PRINCIPLES OF OPTOMETRY

LOCKWOOD

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The Principles of Optometry

AN ILLUSTRATED TEXT BOOK WITH QUESTIONS

For Use in Optical Schools and for Private Students. Especially Designed
to Ground Students in Optometry and to assist in preparing
them for State and other examinations.

BY

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CHAPTER I.

LENSES IN GENERAL.

If we will examine carefully any lens from the trial case we will note, so far as external form is concerned, that there are two surfaces one or both of which is curved, the curve bearing in from the center, or out from the center, as the case may be. Taking these surfaces in combination with a plane surface, or with each other, we will get five distinct lenticular forms, as follows:

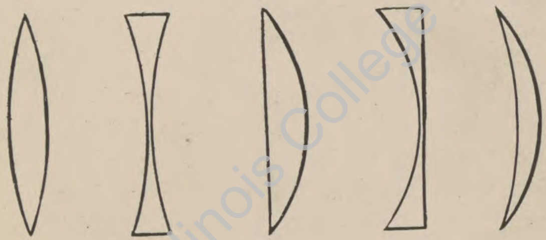

FIG. 1.

These being named in order from left to right: Bi-convex, bi-concave, plano-convex, plano-concave and concavo-convex. This last may have two varieties of form dependent upon which surface is curved the most sharply, the in-curving or the out-curving. In the former case it is often called a minus periscopic, and in

the latter case a plus periscopic. This lens is also sometimes known as a meniscus.

When the sun rises or sets it often seems to give out sharp thin lines of light which were originally supposed to have a material existence. This phenomenon can also be seen by looking at a distant arc electric light, most markedly visible, however, when we look at an extremely minute light of great brilliancy. These lines are called rays. They have no real existence, but are illusions, probably caused by the light passing between the longitudinal fibers of the iris to fall upon the retina as lines of light; provided, of course, that the image of the light itself passing through the pupil is of the right intensity; that is, if the brilliancy is too great, then the eye is dazzled; if not sufficiently brilliant, then the rays do not appear because not enough light comes through the fibers.

It was supposed that these rays were usually invisible; but that for some mysterious reason they might be seen under certain conditions, though no attempt was made to understand these conditions. This theory, for it was nothing more, has long been discarded to be replaced by the wave theory, of which explanation will be made later. Still the term rays of light remains, since by its use the action of light is most readily understood. This makes it necessary to define the word in a way to meet modern requirements, as follows: A ray of light is an imaginary line which any given infinitesimal portion of light wave traverses in joining a source of light with the object on which it falls. It has no dimension excepting length. If we look with one eye at any object the two are joined by the line of sight. This line of sight is identical in nature with a ray of light.

When a ray of light passes from a substance of one optical density to that of another its direction will be changed, pro-

vided it meets the second substance at an acute angle. In Fig. 2 let A represent a ray of light passing through air and falling upon a surface of glass at the point X perpendicularly; then the

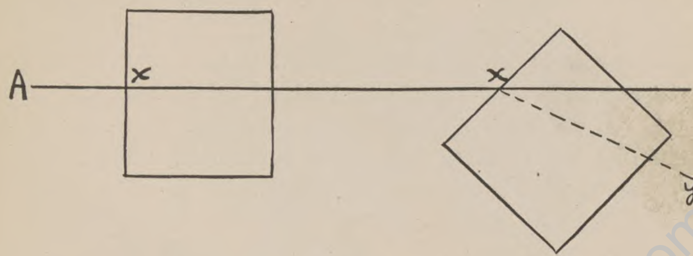

FIG. 2.

direction of the ray will be unchanged, but let the glass surface at this point be inclined then the ray will be bent in toward Y.

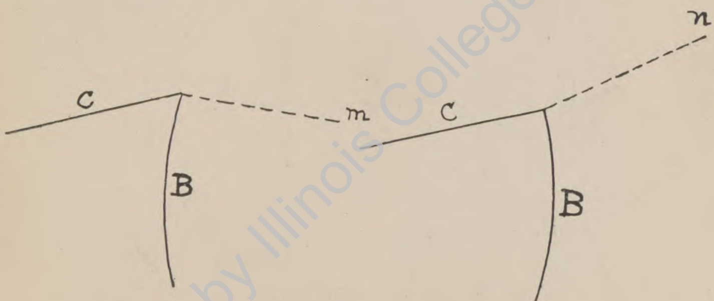

FIG. 3.

It is the working out of this physical law that gives to lenses their peculiar power. Let B in Fig. 3 be a lens surface, and let C be a ray of light. Since C does not meet the surface of B

perpendicularly it will be bent; toward M if the lens surface is out-curving, but toward N if in-curving.

An out-curving lens surface is usually known as a convex or plus surface, and an in-curving one as a concave or minus surface.

The extent to which a ray of light is bent at a given point of a curved surface of a given medium of different optical density depends on the amount of curvature. In Fig. 4 the ray A will be bent in when it meets the curved surface C, but will be bent further in when it meets the curved surface E. The latter has

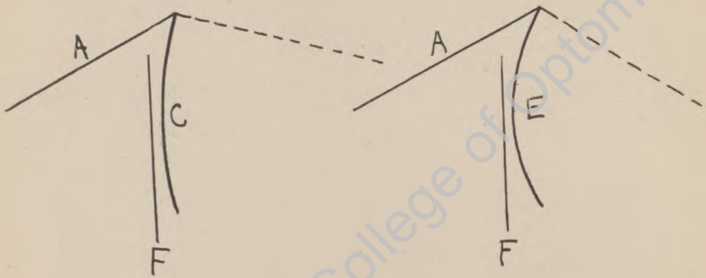

FIG. 4.

the strongest curve because it bends more rapidly away from the straight line F.

When a ray of light meets a lens at an acute angle it is usually bent at both surfaces, but when the ray after passing the first surface has a direction perpendicular to the second surface there will be but one bending.

The bending of a light ray is usually known as refraction.

An incident ray is one which falls on the surface of a dioptric medium.

A dioptric medium is any substance through which light may readily pass; such a substance is said to be transparent.

A refracted ray is one which has been bent at the surface of the dioptric medium through which it is passing.

An emergent ray is one which passes out from a given dioptric medium, at the first surface of which it has been bent.

Parallel rays are those which may be extended indefinitely and always remain the same distance apart. A collection of parallel rays is called a beam, and has its nearest illustration in a locomotive headlight or an electric searchlight.

Convergent rays are those which tend to come to a point. A

FIG. 5.

collection of convergent rays meeting in a point is known as a pencil or cone of light.

Divergent rays are those which separate as they advance. They are the reverse in direction of convergent rays.

Rays of light may be transmitted, reflected or absorbed.

Reflected rays are those which rebound from a polished surface.

Absorbed rays are those which disappear in the substance of the object on which they fall. They usually make themselves manifest as heat.

The principal axis of a lens is a line drawn through its center perpendicular to both surfaces. (See figure 5).

A secondary axis of a lens is any line drawn through its center not perpendicular to both surfaces.

QUESTIONS.

1. Name five styles of lenses.
2. What is the difference in form of a minus-periscopic and a plus-periscopic lens?
3. Which of the two has the thickest edges?
4. Give two other names for periscopic lenses.
5. How may we see light rays?
6. What makes us think that light rays exist?
7. What is the cause of the illusion?
8. Why do we see light rays under some circumstances and not under others?
9. Why do we not see them in ordinary daylight?
10. When a bright light is photographed will it show rays?
11. Give a definition of "light ray."
12. Compare line of sight with light rays.
13. When will the direction of a light ray be changed?
14. When will it not be so?
15. Which way will an out-curved glass surface bend a light ray?
16. Give other names for an out-curving surface.
17. Upon what does the amount of bending of a ray of light depend?
18. Which has the strongest curve, a dollar piece or a tencent piece?
19. What is meant by the term "strongest curve?"
20. When a ray of light passes through a lens at which surface is it bent?
21. Give another name for the bending of a ray of light.
22. What is an incident ray?
23. What is a dioptric medium?
24. What is the difference in meaning between transparent and translucent?
25. What is a refracted ray?
26. What is an emergent ray?
27. What are parallel rays?
28. What is a beam of light?
29. What are converging rays?
30. What is a pencil of light?
31. How can it be made visible?

32. What are divergent rays?
33. What are reflected rays?
34. What are absorbed rays?
35. Into what are these latter usually changed?
36. What is the principal axis of a lens?
37. What is a secondary axis of a lens?

CHAPTER II.

PLUS LENSES.

Plus lenses may be of three forms, regardless of their dioptric power: Bi-convex, plano-convex and periscopic.

That ray of light which passes through the optical center of a bi-convex lens has its direction unchanged; all other rays are bent in varying degree toward the principal axis.

C C, principal axis; A B, incident rays; B E, emergent rays bent toward principal axis.

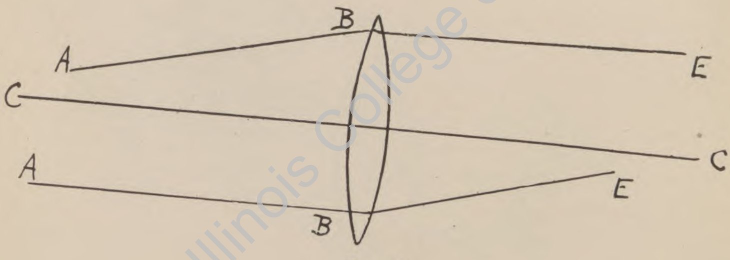

FIG. 6.

A plus lens which brings rays parallel to the principal axis to a point on the principal axis one meter distant is said to have a power of one dioptre.

Let A B, A D and A P be rays parallel to the principal axis of the lens B P which brings all these and intermediate parallel

rays to the point C at a distance of one meter from the lens; then B H is a lens of one dioptre of refractive power.

The point where parallel rays of light are brought to a point by a plus lens is called the principal focus.

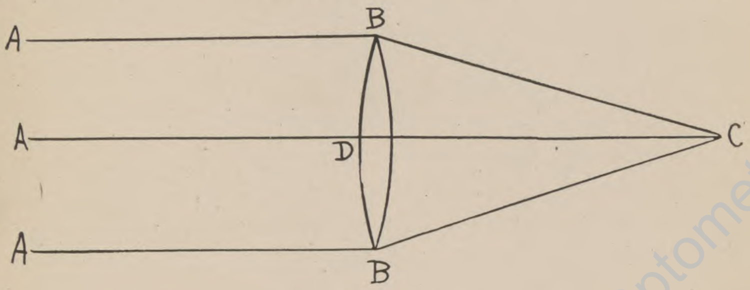

FIG. 7.

The distance of the focus from a lens will vary with the curvature of the lens surfaces, the relative direction of the rays, whether parallel or not, and with the nature of the dioptric medium used. For lenses used in spectacles and trial cases there is a relation between the power of the lens and the curvature of the surfaces approximately as follows: When the curvature of both surfaces of a bi-convex lens is struck on a radius of one meter the lens will have a power of one dioptre. If the curves are twice as sharp—that is, on a radius of half a meter—the power will be doubled. If the curves are on a radius three times as sharp—that is, one-third of a meter—the power will be trebled.

From the above we get the following rule: The dioptric power of a spectacle lens surface is one-half of its dioptric curve. For instance, if the curvature on one side of the lens is on a radius of one-third of a meter (approximately 13 inches) then the dioptric power of this surface is one and one-half, because one-third of

a meter corresponds to three dioptres, and one-half of this is one and one-half dioptres. If the curvature on the other side of the same lens is on a half meter curve; that is, on a two dioptre radius, then its power will be one-half of this, or one dioptre. Add the powers of the two faces together and we have the power of the lens as a whole, in this particular case two and one-half dioptres.

Let Fig. 8 be a periscopic lens, the curvature of one surface on one-fourth of a meter radius, and the curvature of the other surface on a half-meter radius. What is the power of the lens? The first surface being on a four dioptre curve has a power of two dioptres, the second surface being on a two dioptre curve has a power of one dioptre. Add the two values and we have three dioptres of power for the whole lens.

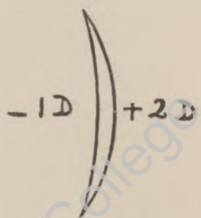

FIG. 8.

QUESTIONS.

1. Name three forms of plus lenses.
2. Which ray of light passes through a lens unchanged in direction?
3. Define a one dioptre plus lens.
4. Define a principal focus.
5. Name three causes which determine the focal point of a lens.
6. What is the relation of curvature of lens surface in a one dioptre bi-convex lens to the dioptric power of the lens as a whole in the case of ordinary spectacles and trial case lenses?
7. Give the general rule in regard to curvature, covering all sizes of lenses of this character.

8. If the curvature on one lens surface of a bi-convex lens is on a 13 1/3-inch radius and the other on a 40-inch radius, what is the power of the lens?

9. In a plano-convex lens one surface is on a curve of 27-inch radius. What is value of lens?

10. In a periscopic lens the plus curve has 2 dioptres of power and the minus curve 1¼ dioptres of power. How far from the lens will the principal focus be?

CHAPTER III.

PLUS LENSES—CONTINUED.

If we will compare a bi-convex lens with a plano-convex lens of the same dioptric power it will be noted that the curves of the former are not so sharp as that of the latter; and this for the reason that all the power of the plano-convex lens must be on one surface, since the other is plano, while in the bi-convex lens there are two available surfaces between which the curvature is divided. In Fig. 9 the two lenses are of the same power, plus four dioptres in each case; the plano-convex lens having it all on one surface, the convex having part on each of two surfaces.

FIG. 9.

Plus lenses have the power of forming images or pictures of distant objects. Hold such a lens with its surfaces at right angles to the line joining it with a distant bright object, such as a church steeple; place a piece of white card or paper back of the lens; move it back and forth until it reaches the right place, when an image will appear clear and exact as to outline,

detail and color, but inverted in every particular—up being down, right being left, front being rear.

Repeat the experiment with light at a definite distance and lens of definite power, and there will be found to be a constant relationship between the distance of the light and distance of the image from the lens. If, for instance, the light, say that of a candle, is sixty inches from the lens, then the image, with a lens of five-inch focus, will be five and five-elevenths inches to the rear. If the candle flame is placed twenty inches from the lens the image will be found at six and two-thirds inches distance. If the light is placed ten inches from the lens the image will be found at the same distance on the opposite side. Now repeat the experiments, but reversing the position of the flame and screen, and it will be found that the two locations in any given case are interchangeable. If a flame at twenty inches makes the clearest image at six and two-thirds inches, then if the flame be placed at six and two-thirds inches the image will be at twenty inches. These interchangeable points are called conjugate foci.

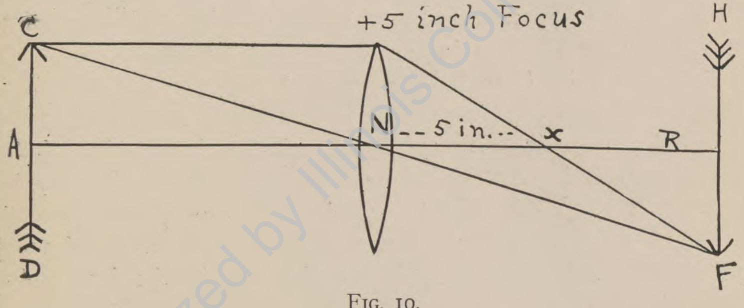

FIG. 10.

The conjugate focus of a lens for a given distant object may be plotted out as shown in the following example:

Let N be a plus lens of five-inch focus, C D a certain object on one side of the lens; where will the image of the object be found on the other side of the lens? Mark the point X five inches from N, because by the terms of the problem N is a five-inch focus lens. Draw the principal axis A R through the optical center of N and at right angles to both its surfaces. From any point of the object C D such as C draw the secondary axis C F, which passes through the optical center of N, hence is not refracted; draw C L to represent a ray of light from C parallel to the principal axis and ending in the lens N at, from which poin t since it is parallel to A R, it is focussed in the point X, which has been marked out as the principal focal point of the lens. Continue this ray o through X until it meets C F in F. This will be the focus of the point C. In the same manner the point H and all intermediate points may be found, from which image may be constructed as shown.

The explanation of this is as follows: Of all the rays emanating from C and falling upon the lens N there is one which passes through the lens unrefracted; this is the secondary axis C F. All the other rays meet somewhere on this axis in one point, and since they all meet in the same point all that is necessary to find that point is to be able to trace any one of those rays. The one selected for the purpose is that which is parallel to the principal axis which must pass through the focal point of the lens since this is what all parallel rays do; hence if a ray be drawn parallel to A R from C and then bent to pass through X and then forward it will sooner or later meet the unrefracted ray from C and the point of intersection will be the focal point required.

The points C and F are conjugate and they are interchangeable, and no other point but C can be conjugate to F, since if C

be brought nearer F will move farther back, and if C be pushed back F will come forward.

Suppose the lens held stationary, its surfaces at right angles to the line connecting the centers of object and image, with the former at a considerable distance away. The focal point may then be shown as in upper cut of Fig. 11. Bring A forward, B will be further back; bring A still further forward, B recedes still further. It will be seen that for every point for A there is a particular point for B no matter how near or far A may be. Notice the increase of size of image.

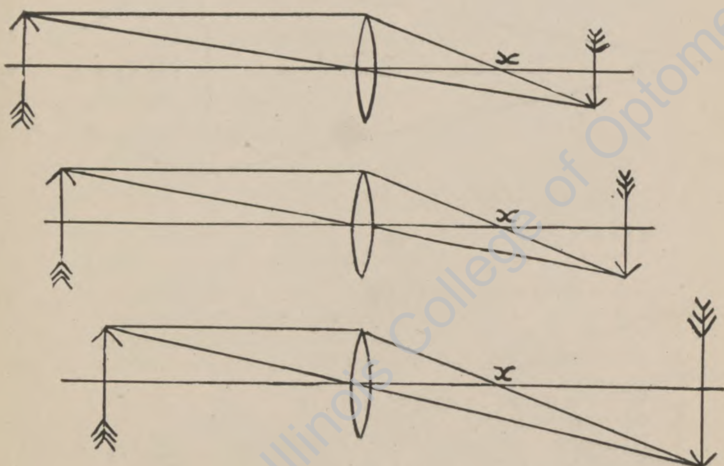

FIG. 11.

The location of the conjugate focus may be calculated as follows: From the dioptric power of the lens subtract the distance of the object in dioptres. The difference will be the distance of the image in dioptres.

In Fig. 12 let the lens be of four-inch principal focus; let the object be twenty inches away; then the image will be at five inches. Figuring a meter as forty inches (which is usual, though the real figure is 39.37+ inches) a four-inch lens has a power of ten dioptres. From this deduct the number of dioptres which correspond to twenty inches, namely, two, and the remainder will be the distance of the image in dioptres which equals a focal distance of five inches.

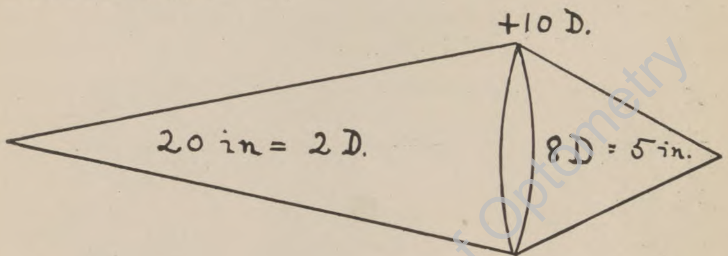

FIG. 12.

QUESTIONS.

1. What is the relation of dioptric power of the surfaces of two lenses each of 4 dioptres, one bi-convex and the other plano-convex?
2. Which of the two requires the lens to be thickest in the center?
3. Compare object and image made by convex lens as to relative position of parts.
4. What is meant by conjugate foci?
5. Give an example.
6. How may a conjugate focus be plotted out on paper?
7. Explain why this method is correct.
8. What is the effect in regard to image of bringing an illuminated object closer and closer to a lens (but kept outside the principal focus of the lens)?
9. Give the rule for calculating the focal point, with the power of the lens and the distance of the object stated.
10. How many inches in a meter?
11. What is usually accepted and why?
12. What focus equals 1.25 D; 2.25 D?
13. How much dioptric power has a lens of 6-inch principal focus?

CHAPTER IV.

If the screen upon which the image formed by a plus lens is received be removed the clear image will disappear. It is still there, however, or rather the intersection of rays which form it, and is now known as an aerial image, invisible it is true, but ready to show itself when a screen is supplied.

A diffused image is one in which details are blurred. This is always formed when the screen is out of focus. An object which is a source of light may be assumed to be made up of innumerable points, the light from which must after passing the lens come to a point again if the image is to be a clear one; otherwise the image will consist of an infinite number of small

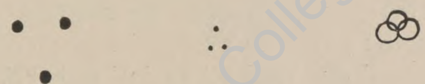

FIG. 13.

overlapping circles due to the fact that the light rays have either not yet met in a point, which will be when the screen is too near the lens, or that they have met and crossed, which occurs when the screen is too far removed. The image caused by these overlapping circles is a blurred or diffused one.

In Fig. 13 let the source of light be a number of dots as shown at A, then the image at the focus of the lens will be as at B, but if the image is out of focus it will appear like C, either because the

rays have not come to a point or else because they have done so and then have crossed.

A focus is said to be virtual when the lens forms no image, but would do so if the direction of the light waves were reversed. In Fig. 14 let A B represent a plus lens and C an object

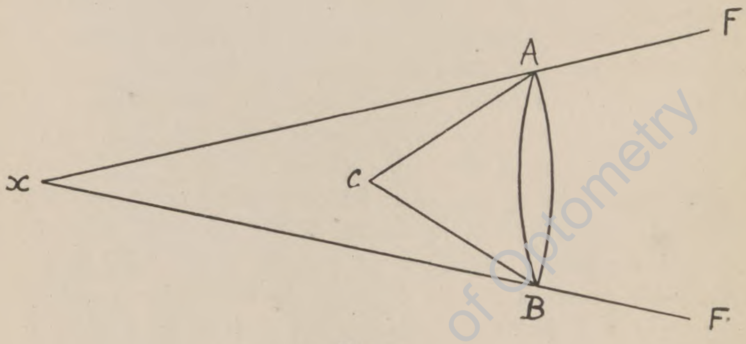

Fig. 14.

closer than the principal focus. In this case the lens has not sufficient dioptric power to render the rays even parallel, hence they diverge as shown by the lines A F and B F. Trace the lines backward till they meet and the point of intersection will be the virtual focus as shown at X.

In looking through a magnifying glass at a small object we do not actually see it, but rather a virtual image of it lying further back. For instance, we hold the eye ten inches from a small object and can see it distinctly as of a certain size. Now interpose a lens of two-inch focus at a distance of one and one-half inches from the object. We will see a virtual image four times as large six inches behind the glass. Bring the eye down to four inches from the lens thereby making the virtual image ten inches

away, and the object will be magnified four times. Let Fig. 15 represent the two conditions.

By four times is meant four diameters, or sixteen times as much surface.

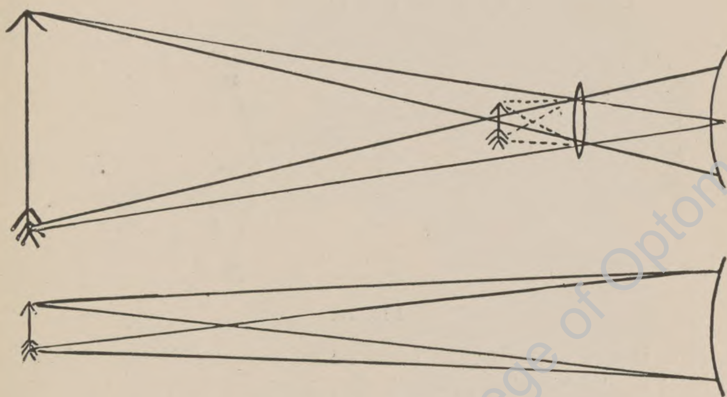

FIG. 15.

A virtual focus or image is sometimes called a negative one, especially when made by minus or negative lenses.

The location of a virtual focus may be plotted out. Draw from any point of the object not on the principal axis two lines, one a secondary axis, the other at first parallel to the principal axis till it reaches the lens, then backward through the principal focal point on the same side of the lens as the object until it meets the first line. The junction point is the virtual focus desired.

This method is practically the same as that employed in finding actual foci, as previously shown; and is explained in the same manner.

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In Fig. 16 let A B be a plus lens, let C D be the object; then from some point of the latter such as C draw a secondary axis; also draw a line from C to the lens parallel to the principal axis until it meets the lens, where it is bent so as to pass through the principal focus S. The point of intersection will be at the arrow tip back of C, hence the focus is a virtual one.

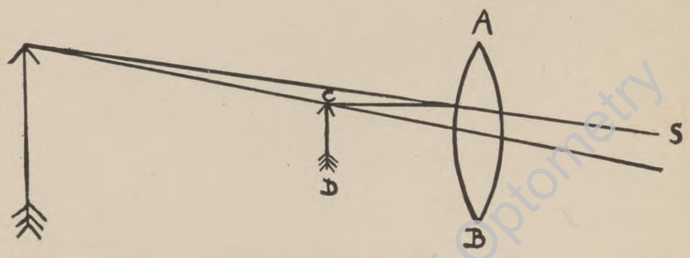

FIG. 16.

This is not a conjugate focal point, because if the object be changed to the location of the image its focus will not be at C D, as may be discovered by taking some point of it and plotting out the focus, which in the case of Fig. 16 will be found somewhere back of S.

That the difference between a virtual focus and a conjugate focus may be quite clear it is to be noted that the former is always on the same side of the lens as the object, while the latter is on the opposite side; also that the former cannot be caught on a screen because it has no actual existence and is erect, while the latter can be caught on a screen and is inverted.

The rule for getting a virtual focus by calculation is the following:

Figure the distance of the object from the lens in dioptres. From this subtract the dioptric power of the lens. The difference

will be the distance in dioptres of the image. Change this latter to inches to get the required answer.

Let A B, Fig. 16, be a lens of four-inch focus (that is, of ten dioptres), and let C D be two and one-half inches from the lens (sixteen dioptres); then the image will be at a distance of six dioptres, which equals six and two-thirds inches.

When the rays of light from a given point pass through two lenses in succession the size and distance of the image will vary with the power of the two lenses, the distance of the object and their distance apart. The farther away the object the smaller the image will be, while the farther apart the two lenses within certain limits and the weaker their dioptral powers the larger the image.

In all cases the relative sizes of object and image follow regular rules, and can easily be calculated. Supposing only one lens is used, then the size of the two is directly as their respective

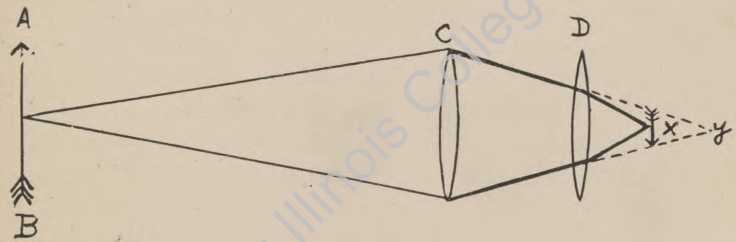

FIG. 17.

distances from the lens. When image and object are equidistant then they will be found to be of exactly the same size; when the image is one-half as far away as the object it will be half the size; when it is twice as far it will be twice the size, etc.

To figure out the size of the image where double lenses are used first figure what one lens would do as regards the size and distance of the image and then consider this latter as an object and see what effect the second lens will produce in the way of an image. In fact, this method may be continued through any number of lenses in succession.

Let us suppose that the object is two meters away from the first lens and that there is a space between the two lenses of seven inches, and that the power of one lens is two dioptres (twenty-inch focus) and the power of the second lens is four dioptres (ten-inch focus), then the question is to calculate the distance and size of the image.

Let A B, Fig. 17, represent the object, C the first lens and D the second one; then if the second lens were not present the image should be at Y, twenty-seven inches (one and one-half dioptres) from C, but since C and D by the terms of the problem are seven inches apart then D must be twenty inches from Y, or two dioptres, and the lens D being four dioptres, the distance to X, the focal point of both lenses taken together, will be six dioptres, or a distance of six and two-thirds inches. Now as to its size. The potential image Y being twenty-seven inches from C while A B is eighty inches it must be twenty-seven eightieths as large as A B, and since the potential image at Y is twenty inches from D while X is six and two-thirds, then X will be six and two-thirds twentieths of the size of Y, or nine-eightieths of A B, which is the answer required.

$$\frac{27}{80} \times \frac{6\frac{2}{3}}{20} = \frac{27}{80} \times \frac{20}{60} = \frac{9}{80}$$

QUESTIONS.

1. What is an aerial image?
2. How may it be shown to exist?
3. What is a blurred image?
4. Give an example.
5. What will be the form of the image if the light be a point and the screen be too near the lens?
6. What will be its form if the screen be too far away?
7. What is a virtual focus?
8. Under what circumstances will a virtual focus be formed by a plus lens?
9. When we look through a magnifying glass at a small object, what do we really see?
10. Why does the object under these circumstances look magnified?
11. What is a negative focus?
12. What two lines are necessary in plotting out a virtual focus?
13. What is the difference between a conjugate and a virtual focus?
14. With what two lines can we plot out the location of a virtual focus?
15. Give the rule for calculating the location of a virtual focus.
16. Give an example with a plus two and one-half dioptre lens.
17. In the use of double plus lenses, what factors determine the size of the image?
18. At what distance from a plus lens are the object and the image of the same size?
19. At what distance must an image be from a plus lens when the object is one-half its size?
20. In figuring the effect of a double plus lens, how do we proceed?

CHAPTER V.

If a plus lens is held a short distance in front of the eye and then moved forward the distant objects seen through it will seem to increase in size. This is due to the fact that such a movement forward causes the retinal image to increase, and this we interpret as a growth in the object. This will be made clear by Fig. 18. Let A B be the object, C the lens, and R the retina. With the lens held as shown; that is, close to the eye, the two will act as one lens and the image will be at Y and of a certain size. Move the lens forward and the power of the couplet decreases as will be explained later; but a decrease of dioptric power always means an increase in the size of the image; hence the eye will see the object grow. This appearance of growth is accentuated from the fact that the image in the second case will be at Y (in the second cut), which will cause a diffused retinal image which will be still larger.

At first inspection it will appear as if the image at Y should be smaller as shown in the second cut than in the first because formed closer to the cornea, but this is not so, because Y is not an image of the eye alone, but of the eye and C together and if the size of images be figured out Y in second cut will be considerably larger. In comparing the two drawings of Fig. 18 it will be noticed that the forward movement of C is much greater than the forward movement of the retinal image. In practice a one-dioptre lens will make an apparent increase in size of 100 per

cent. when the lens is moved back and forth to the full reach of the arm. With a plus one-fourth dioptre lens the growth will be much less but still well marked.

If distant objects be viewed through a plus lens and the latter moved from side to side the distant objects will seem to move in the opposite direction. This is due to the fact that as the lens is moved the retinal image of the distant object also moves and

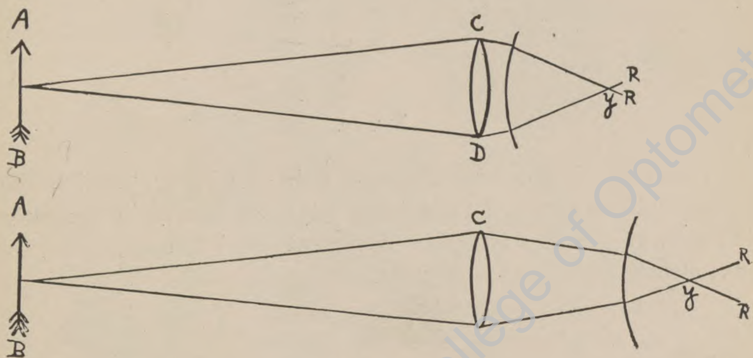

FIG. 18.

in the same direction, hence the eye seems to see the object move the other way. In Fig. 19 let A, B and C be three objects and let D be an eye, then it will be seen by the lines traced that a retinal image is always opposite, as to location, to the place of the object.

The eye is not cognizant of any movement of a retinal image, but since the upward motion of an object produces a downward movement on the retina, therefore when we in any way cause the latter the mind interprets the phenomenon in the usual way, namely, that the object moves the other way.

In Fig. 20 let A be a distant object, B the lens to be tested and C a strong lens to correspond to the dioptric media of the eye; then with B in the upper position the image will appear on

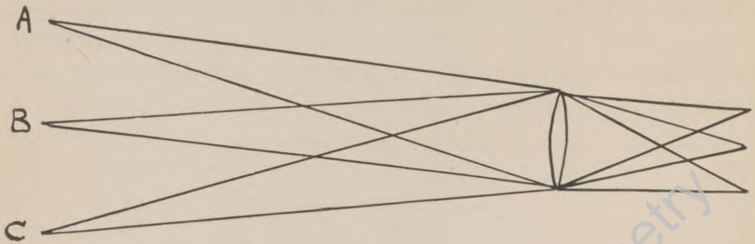

FIG. 19.

the retina at X, but with the lens B in the lower position the image will have moved to Y; that is, it has moved in the same direction as the lens and this movement the brain sees reversed or contrary to the lens movement.

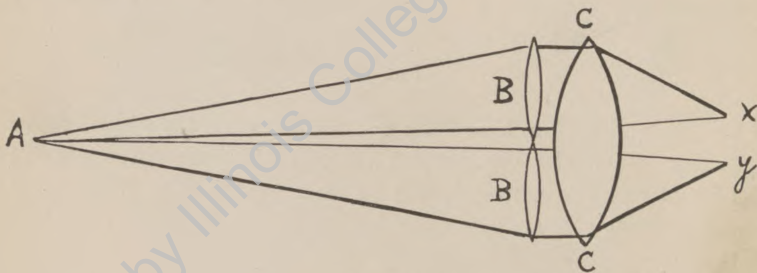

FIG. 20.

In order to find the dioptric value of an unmarked plus lens any of the following methods may be employed:

First—Hold the lens so that the rays of light from the sun

may strike it at right angles to its surface; move a piece of white paper back and forth on the opposite side of the lens until the image formed thereon is the most clearly defined. Measure the distance in inches of this image from the lens and express the same in dioptres. For example, if the image be at a distance of five inches the lens will have a power of eight dioptres. (See Fig. 21.)

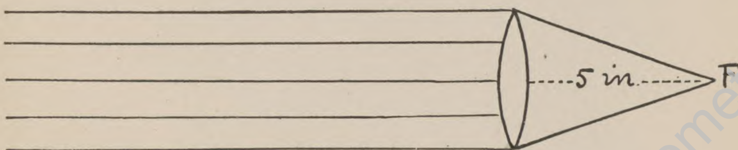

FIG. 21.

Second—Hold a lens in such a position in reference to a candle flame or other convenient source of light that the image and the object will be equally distant from the lens; half of this distance expressed in dioptres is the required value. In Fig. 22 let A be the object, B the image. Take one-half of the distance from either the object or image to the lens.

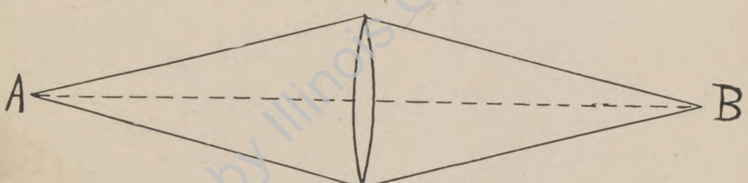

FIG. 22.

Third—Place the source of light in any convenient position; find focus of lens; express the distance from lens to light in dioptres; also the distance from lens to image in dioptres, and add the

two results. In Fig. 23 let A be the object, B the lens and C the image. If the distance from A to B is eighty inches; that is, one-half dioptre, and the distance from B to C is twenty inches or two dioptres, then the value of the lens will be two and one-half dioptres.

Fourth—A lens measure may be used. Apply this instrument to one surface and read off the dioptric power; repeat the process with the other surface, and add the two results. For

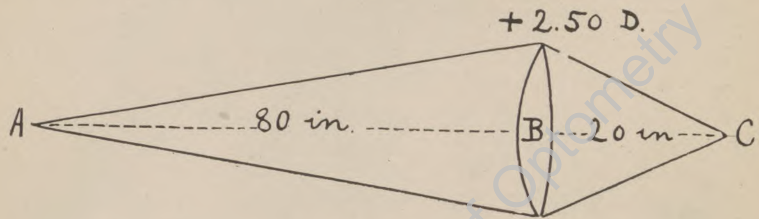

FIG. 23.

instance, if one side reads +2 dioptres and the other reads —1.25 dioptres, then the sum of the two will be +.75 dioptres, or a focal distance of fifty-three inches.

Fifth—The unknown lens may be neutralized from the trial case. For this purpose hold a minus lens in front of and in contact with the plus lens to be measured; move the two from side to side before the eye. If distant objects appear stationary the two lenses are of equal power but of different signs. If the distant objects seem to move another minus lens should be tried until the desired result is attained. Very exact measurements can be made in this way correct to one-eighth of one dioptre.

To test a lens for optical correctness notice carefully the image which it forms. If this is not clear and distinct either the surface is more or less irregular in curvature or different portions

of the lens have different refractive powers. It must, however, be noted in this connection that for lenses of high dioptric power the image is never so good as for lenses of medium or low powers. This is due to spherical aberration to be explained farther on.

To test a lens to see if the surfaces are without scratches, polish well with a clean soft cloth and hold so the light may strike it in all possible directions, when all flaws and scratches may be made out. Sometimes these will come out better if the tips of the fingers are first passed gently over the surface, as in this case a quantity of oil is left in the cracks which renders them visible.

QUESTIONS.

1. Why does a distant object, seen through a plus lens, seem to grow in size as the lens is pushed forward?
2. Which makes the largest image, a two dioptre or a four dioptre lens?
3. Which produces the largest image, two plus lenses close together or the two lenses separated?
4. As a plus lens is moved from side to side, how do distant objects seem to move?
5. What is the reason for this?
6. How does a spot of light, moving from right to left on the retina, seem to move on the opposite wall?
7. Why is this?
8. How can we tell the focal distance of a plus lens by testing with a far distant source of light?
9. Give two other ways of reaching the same result with the light near by.
10. Give some examples.
11. Describe the method of testing by means of a lens measure.
12. How would you proceed to neutralize a plus lens?
13. How would you tell whether the lens is of first-class quality?
14. How would you examine a lens to discover flaws and scratches?

CHAPTER VI.

To find the optical center of a lens proceed as follows: Draw on a piece of cardboard or paper a long straight line; hold the lens to be tested between this line and the eye; move the lens about until looking at the line over the lens, under the lens and through the lens, all at the same time; the line appears unbroken; mark the location of this with ink, and turn the lens at right angles and repeat. Where the two ink lines cross will be the optical center. This should be identical with the geometrical center, unless it has purposely been decentered. Fig. 24 shows the appearance of the line in both cases.

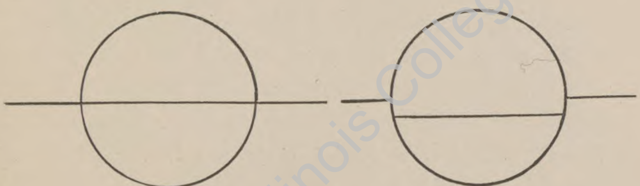

FIG. 24.

This same test may be more quickly made as follows: Instead of one long line on the paper draw two, one at right angles to the other. Hold the lens between the eye and the intersections until both lines appear unbroken. The crossing point will then mark the optical center. Fig. 25 shows three lenses, one in which

the optical and geometrical centers coincide and two in which they do not.

There are two methods of numbering lenses, one approximately by the length of their foci in inches, the other by the dioptric method. The latter is almost universal among refractionists, though the former is in common use. In eye testing it often happens that the operator wishes to know the effect of

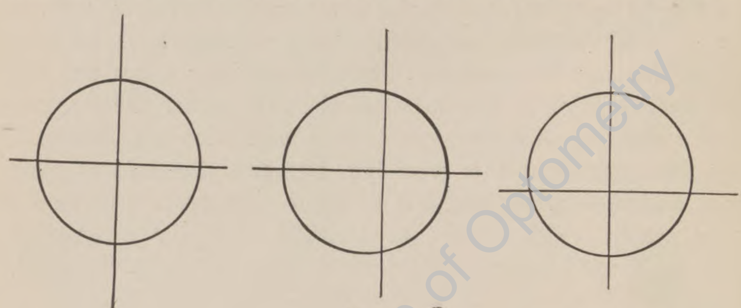

FIG. 25.

slightly increasing the power of the lens before the eye. To learn this he takes from the trial case a low-power lens and holds it first before and then away from the lens already in place. Let us suppose that the lens already in place is a No. 32 and the low-power added is a No. 160, which is the power of the two combined? It cannot be the two numbers added together, since a No. 192 is of lower value than either of the others. To solve the problem we must add the reciprocals of the powers, which will give us the reciprocal of the answer required. Therefore in this particular case we would proceed as follows:

$\frac{1}{32} + \frac{1}{160} = \frac{5+1}{160} = \frac{6}{160}$, and dividing 160 by 6 we have for

the answer 26 2-3, to which the nearest number of lens is a No. 27.

By the dioptric method the mathematical work is much easier. We simply add the two values together, for instance, if the first lens is a +1.25 dioptre and the second is a +.12 dioptre, the value of the two together will be their sum, or +1.37 dioptres.

So far it has been assumed that all the rays from a given point which fall upon the surface of a given lens will be refracted exactly to one point. This, however, is only true for extremely small lenses which theoretically may be considered as having no thickness. In practice a phenomenon is encountered which is called aberration. Make a screen of cardboard with a central hole about one inch in diameter so arranged that a lens may be held over this hole while the rest of the screen causes a shadow to fall upon the place where it forms an image. Place over the

FIG. 26.

central hole a lens of convenient power, say, five dioptres. Over this lens place a cardboard cover pierced by a single pin-hole. Arrange a candle several feet away and then catch its image on a piece of white paper a little over eight inches to the rear of the lens. This image is formed by the rays of light which after traversing the lens then pass through the pin-hole. Note that this image is marvelously well defined, but not brightly illu-

minated. Now remove the pin-hole screen and the image will become very much brighter but less clear as to outline; in fact, with a large strong lens it may be quite blurred. Its illumination is greater, but at the expense of clear outline, or to use the proper word definition. By placing covers with holes of different sizes, one will be found in each case which gives the most satisfactory result. This will be a compromise between illumination and definition. Both of these are important in vision. We must have both; but how much of each to get the best result, in those cases where we must choose between the two, varies with the light and the individual.

The reason why an image made by a lens covered with a pin-hole diaphragm has better definition than one made with the entire lens is because the central part of the latter and its outer portions do not equally refract light; that is, the peripheral rays do not come to the same focal point as the central ones. This variation in the location of the focus is known as spherical aberration; that is, a wandering of the rays due to the spherical form of the lens surface. This trouble is not very prominent in the practice of refraction, since the amount of lens surface used is limited by the size of the iris opening, but with stronger lenses it is a serious obstacle. The effect of the iris opening in limiting the amount of lens surface used may be seen in Fig. 26, where A is a plus lens, and B is the eye. It is evident that but a small portion of the lens A can be in use at one time.

Spherical aberration causes much trouble in the case of optical instruments. Fig. 27 shows a telescope reduced to its two simplest elements. A is the field lens and B the eye-piece. An aerial image of the object is formed at C, which is then examined by means of the lens B, which latter, acting with the dioptric media of the observing eye, causes an image to be formed on the retina

much larger than the one formed there without the telescope.

One of the factors in good vision is size of retinal image, as this causes detail to come clear which otherwise could not be made out; but in order that this detail may be visible its outlines must be clearly defined, and this is where spherical aberration limits the usefulness and practicability of optical instruments. To

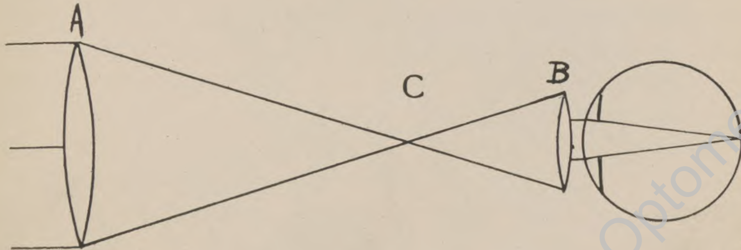

Fig. 27.

obviate the difficulty of lack of definition, the aperture of the lens may be made small, but this reduces the amount of illumination of image, so that again the detail is missed for lack of light.

QUESTIONS.

1. How can the optical center of a lens be found?
2. What are the two methods of numbering lenses?
3. Describe the old or inch system?
4. What is one advantage of the new dioptric system?
5. Give an example.
6. Which is nearest to the lens, the focus of the central portion or that of the periphery?
7. How can this be proved?
8. Describe the difference between the image formed by the entire lens and one in which the rays must also pass through a pin-hole diaphragm.
9. Under what conditions does an object make the most satisfactory image?
10. What is spherical aberration?

11. What is the effect of the iris opening of the eye?
12. Describe the simplest form of telescope.
13. How would you proceed to make such a telescope from trial case lenses?
14. What three physical factors are necessary to clear vision?

CHAPTER VII.

Let us suppose that the image formed upon the retina by a distant object when looked upon by the eye alone were to be of a certain size and brilliancy. Now if the eye were twice as deep while the pupil remained the same size then there would be a retinal image of twice the diameter, but since it would be caused by exactly the same amount of light, as determined by the size of the pupil, it would lose in brilliancy what it gained in size, in which case its detail might be less satisfactorily seen than in the first case, or, in other words, the gain in size would be more than offset by the loss in illumination. A telescope produces exactly this effect; hence it must be made to admit more light, and it is here that spherical aberration causes trouble. In Fig. 27 it will be seen that the amount of light entering the eye is very much more when it comes by way of the field lens and eye-piece as shown than when these are removed. The amount of light which goes to make up an image depends on the angle of aperture. The angle of aperture is measured by the difference in direction of two lines, one from any given point in the object to one extremity of the lens, and the other from the same point to the other lens extremity. In the naked eye this is settled by the width of the pupil; in a telescope by the size of the objective, the latter having by far the greater area of the two.

It should be noted here that angle of aperture and visual angle are not the same, the latter being the angle formed by lines drawn

from the extremities of an object to the center of the crystalline lens.

Since for a brighter retinal image there must be an increased angle of aperture, and since this in turn means rapidly increasing spherical aberration, it follows that with ordinary lenses the effective satisfactory magnifying power must reach a limit which can only be extended by in some way getting rid, completely or partially, of the spherical aberration.

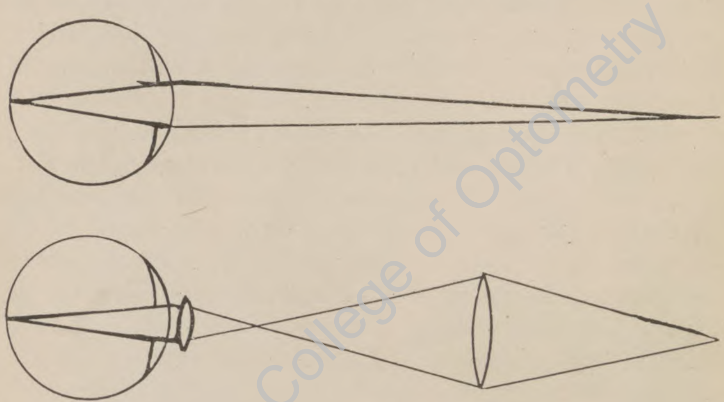

FIG. 28.

Fig. 28 will show the difference in angle of aperture of an object viewed through a telescope and by the naked eye.

If we look through an opera-glass, and then remove the glass from before the eye, we will notice that the objects viewed are not only larger but much brighter. This is due to the large size of the object glass in relation to the relatively low magnifying power, and shows that with this form of instrument the angle of aperture may be increased disproportionately to the magnifying power.

On the other hand, if we look through a telescope in the same way we will find the object much more magnified but the field of view is duller. In the latter case the limit of satisfactory magnification has been reached. If we wish to go further we must increase the diameter of the objective, but in so doing we must also get rid of the spherical aberrations.

One of the greatest problems for the mathematical optician to solve has been to decrease spherical aberration. This in theory may be done in two ways, one by making the curve of the lens such that rays from its entire surface will meet in one point. This in practice is impracticable and economically impossible. The necessary curve may be figured out, but even if the machinery to grind such a curve were possible to make, the cost would be prohibitive. The other way to get rid of the spherical aberration is to make compound lenses some of one optical glass and some of another, and to so arrange them in relation to one another, with due regard for optical density, curvature, distance apart, size, power and thickness, that the desired end may be reached. This line of work is the most difficult branch of mathematical optics, and it will be sufficient to say here that while there is no combination which eliminates all the difficulty, still the experts have come very near to absolute correctness; and it is for this reason that the efficiency of optical instruments has so much increased in this generation.

Spherical aberration may show itself in the human eye to such an extent that vision may be affected. Glasses will in this case produce little or no improvement. To remedy it there are two remedies, each inconvenient; one to instill a drug in the eye which will keep the pupil small, or having the patient wear a pair of pin-hole discs. The object in each case is the same; the rays which cause the blurring, namely, the peripheral ones, are to be cut off.

There exists another form of aberration, called chromatic aberration, which also interferes with clear definition. It is due to the fact that the various colors of which light is made up are not equally refrangible. All of the colors, or rather the waves which cause them, pass through space with the same speed, but this is not the case with glass, or, for that matter, any material substance. The order in which the rays are refracted is always the same in the case of any particular medium, but the extent of the difference between any two colors varies with the kind of glass; that is, with its optical density. The red is the least refrangible, the violet the most, and the yellow midway. This means that in case of a plus lens the violet rays will have the shortest focus, the red the longest, and the yellow between the two; that is to say, the more slowly any class of waves moves in passing through the glass the shorter the focus where they meet, and the quicker they pass through the farther away they come together.

Assuming that the yellow rays are properly focused, there will overlie each yellow spot a diffused violet image, because the rays of this color are focused in front and have diverged, and there will also be an overlying area of red due to the red not yet coming to a point. Every infinitesimal point of the object will be duplicated in this way in the image resulting in a confusion of detail. This effect is the more pronounced the stronger the lens, but since in powerful optical instruments strong lenses must be used, if follows that this form of aberration causes considerable trouble. Here again is the province of the mathematical optician. His method is the same as when he is dealing with spherical aberration; the use of such a combination of lens as will approximately cure the trouble. A lens made for this purpose is called achromatic. The usual way of making it is to have one lens of crown glass, bi-convex, of any desired power, and then to cement to this

a plano-concave lens of half the dioptric power. The extent to which the red and violet rays are diffused is twice as much with flint glass as with crown glass, hence a negative flint glass lens of a certain spherical or dioptric power will, so far as dispersion

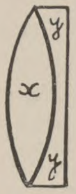

FIG. 29.

is concerned, balance one of twice the dioptric power when made of crown glass; hence the combination will be achromatic and still have considerable dioptric power. This is shown in Fig. 29, where X represents a crown glass plus lens of any power and Y represents a flint glass minus lens of half the power, the result being an achromatic plano-convex couplet equal in power to one-half of X.

The property of a lens to bring the different colors of white light to different foci is called its dispersive power.

In rare cases the human eye may be troubled with chromatic aberration, in which cases objects will appear fringed with colors. There is no known cure.

It is because of the expert work done by mathematical opticians that so much scientific progress has been made in microscopy, spectrum analysis and other branches of optical science.

QUESTIONS.

1. Suppose an image be magnified with no increase in light: what is the reason that it may not be satisfactory?

2. Why are telescopic images apt to be dim?
3. How can this be corrected?
4. What is the angle of aperture?
5. What is a visual angle?
6. When more light is admitted to a telescope, what annoying condition comes with it?
7. Why do objects seen through an opera glass appear both larger and brighter?
8. Why is not this the case with an ordinary telescope?
9. Name two ways in which spherical aberration may be corrected.
10. Which one is practicable?
11. How may spherical aberration in the human eye be corrected?
12. What is chromatic aberration?
13. What rays are most refrangible?
14. What rays are least refrangible?
15. Which has the shortest focus and why?
16. What is the effect of chromatic aberration?
17. Describe an achromatic lens.
18. What is the dispersive power of a lens?

CHAPTER VIII.

If a light-proof box be made and a pin-hole, or rather a needle-hole, be drilled through one of its sides, an image will be formed on the opposite inner wall without the use of a lens at all. This image will be soft in outline, but distinct. The following is the explanation of its formation. (Any given object is made up of innumerable points, each of which radiates light in all directions.) Some of these pass through the needle-hole to the opposite inner side of the box, and there become visible as a minute area of the same color as the point from which it comes. It is the aggregations of these minute areas that make up the image as a whole; the image, in fact, is a blurred one, but the amount of the blurring is so slight, due to the smallness of the hole, that the detail of form and color and relative position is retained, though the picture is faint because of the small pencils of rays which form each point of it. This is because of the small angle of aperture.

In Fig. 30, let A be a series of dots, and let B be a light-proof box, with a needle-hole at the point C. Now such portion of the light from the upper dot as meets the needle-hole will pass through and appear at the point M; of course of the same color. The same will apply to the light coming from the other dots, with the result that there will be an image of the object, but reversed. If the upper dot of the object be of a red color, then the lower dot of the image will be also red; if the middle dot of the object be yellow, then the middle dot of the image will be the same, and

finally the bottom dot of the object and the upper dot of the image will agree. All this may be traced out as shown by the lines in the figure. As to the relative size, this will depend on the relative distances in accordance with geometrical laws.

Pin-hole photography is done in this way. The plate has to be exposed for a considerable time, but because of the softness of the lines the picture will be an artistic one, which would also be the case were there a lens before the needle-hole of such

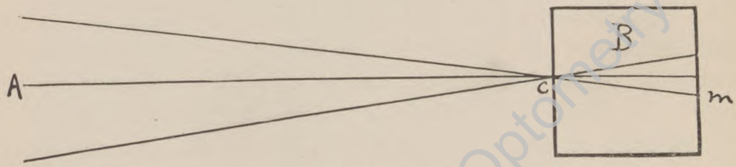

Fig. 30.

power that the location of M would be out of focus.

The science which deals with the functions of triangles is known as trigonometry. Optical refraction is based upon it, and so far as this is the case, it will be taken up here.

It was noticed ages ago that a stick held in water appeared broken at the surface, the amount of the break depending upon the angle it made with the surface of the water. The relative positions of this angle and the one made by that portion of the stick which was under water in any given case was always the same; that is, each angle of stick with the surface was accompanied by another angle of stick beneath the surface which was invariable. These angles had been measured many times by philosophers who tried to discover the rules which governed their formation but without success. Finally the rule was made clear through the trigonometrical relation of the angles the stick made with a line

drawn perpendicular to the surface of the water. The result was the law of the index of refraction. This in reality is the relative speed of light through any substance as compared with its speed through space. This latter is usually accepted as the same as its speed through air, though there is a trifling difference in the rate. On the basis of trigonometry the index of refraction of any substance is expressed by dividing the sine of the angle of refraction into the sine of the angle of incidence in air.

The difference in direction of two lines is called an angle. It is not measured by the shortness or length of the lines, but by their difference in direction expressed in degrees. To find the number of degrees describe a circle through the line with their

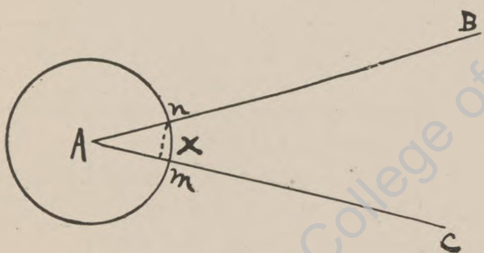

FIG. 31.

junction as a center, and whatever number of degrees are cut off by the lines is the value of the angle. There are three hundred and sixty degrees in a circle. In Fig. 31 let A B and A C be the two sides of the angle meeting in the point A; describe any circle on A as a center; divide the circumference into three hundred and sixty equal parts; count off how many of these are cut off; or find what part of the circle X is and multiply by 360. For instance, if X is one-sixth of the circle then the angle is sixty

degrees. It is evident that the result will be the same, no matter what the size of the circle may be.

If from the point N a perpendicular be drawn to the radius of the circle A M, this will be the sine of the angle at A. The same will be true if the perpendicular be drawn from M to the line A N. Expressing this in the form of a definition, the sine of an angle is a perpendicular drawn from one of the sides of the angle to the other.

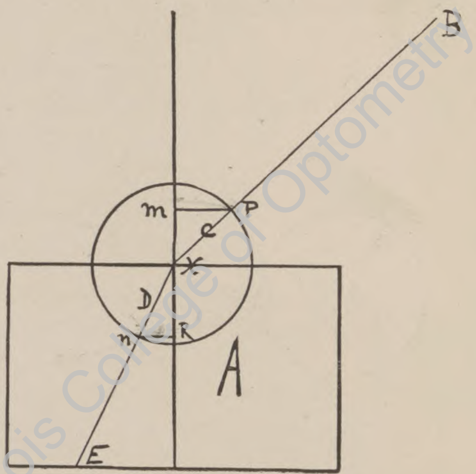

FIG. 32.

In Fig. 32 let A represent a block of glass, B C be a ray of light meeting the surface of the block at X, and let D E be the direction of the ray after refraction. At X draw a line perpendicular to the surface, and with X as a center describe a circle. Now draw the sines M P and N R. Measure the length of the sine N R and divide it into the length of the sine M P. The

quotient will be the index of refraction for this particular quality of glass.

Suppose N R is 2 inches and M P is 3 inches; then the index of refraction is 1.5, which will always be exactly right, no matter at what angle B C may meet the surface of the glass. From this we may make another definition of index of refraction; namely, the relation of the sine of the angle of refraction to the sine of the angle of incidence. Note that these angles are figured from a line perpendicular to the surface and not from the surface itself. This law was discovered by Snellius in 1671.

Every substance, so far as known, has its own index of refraction, which is low for gases and high for liquids and solids. For instance, the index for air is scarcely different from space, while water is 1.33, crown glass 1.52, the lens of the eyes 1.41, etc.

Not all of the light falling upon the surface of a dioptric medium is refracted; part is absorbed and disappears and part is reflected. Glass reflects 25 parts out of 1,000; mercury 666 out of 1,000, and polished silver 920 out of 1,000. Under certain circumstances no light will be refracted, but all will be reflected. The angle at which this occurs is called the angle of total reflection.

A ray of light, after being refracted at the first surface of a dioptric medium, may meet the second surface at so great an angle to the perpendicular that when we multiply its sine by the index of refraction the answer will be more than the radius of the circle. Since the radius of a circle is the highest possible sine that any angle can have, therefore the ray will not be refracted at all but will be totally reflected. The angle at which this occurs is called the angle of total reflection. In glass it is about forty-two degrees.

QUESTIONS.

1. Explain the image made through a pin-hole.
2. Why is such an image not badly blurred?
3. Is such an image erect or inverted, and why?
4. Describe pin-hole photography.
5. What is trigonometry?
6. What is meant by refraction, and give an example.
7. How is the index of refraction calculated?
8. What is an angle?
9. How measured?
10. What is the sine of an angle?
11. Under what circumstances would the index of refraction figure out 1.75?
12. Give a definition of index of refraction.
13. Who discovered the law of the index of refraction, and when?
14. Give the index of refraction of a number of substances.
15. Why can we see ourselves in a piece of plain glass?
16. What is meant by the angle of total reflection?

CHAPTER IX.

Wave Theory and Rapidity of Light.

A shadow as ordinarily defined and understood is an area of darkness produced by cutting off the illumination from a surface by a suitable opaque screen, but this is strictly speaking a light shadow. There are other varieties of shadows such as water wave and sound shadows, the nature and the cause of which are in a sense identical with the nature and cause of light shadows. Drop a stone into the center of a pond of still water. A wave will at once be formed, visible to the eye, circular in form, travelling outward from the center of the disturbance in an ever-widening circle, the wave itself decreasing inversely as the diameter of the circle, until if nothing interrupts its advance, it dies away upon the shores. Should this circular wave meet a small fixed obstacle, such as a stake, it will run around it, and the two broken ends will reunite on the other side. Still close examination will show a very small area of still water directly behind the stake inside of the point where the broken ends of the wave meet. This area of still water is a shadow. Make the obstacle larger, such as a breakwater or an anchored ship, and the shadow is much more extensive. Comparing the still water as described above with the shadow cast by an opaque screen we can get the following definition:

A shadow is an area of quiet in the path of any wave motion caused by some obstacle.

According to the old theory, light was caused by the visible object, continually throwing out infinitesimal particles of matter which bombarded the surface of the eye and then passing through its media reached the retina there in some way beyond human comprehension, to be interpreted as the sensation of light. In view of the enormous velocity of light, 186,000 miles per second, it was quite evident that these small particles of matter must have little weight, or else the eye would be injured; and since there was no apparent injury of this kind, or even the slightest recognition of the bombardment, it was accepted as correct that this matter was without weight, or, as the scientists expressed it, imponderable. This was the belief held by Newton and his reason is curious, for, says he, how else can light reach us; it cannot be any form of wave motion, else shadows would not exist, which shows very conclusively that the great philosopher did not know as much about the nature and formation of shadows as he should. The force of Newton's authority was so great that the acceptance of the wave theory of light was delayed for many generations.

Newton was in error. Light does run around an obstacle and meet on the opposite side, but its rapidity is so enormous that by the time the meeting takes place the light wave is thousands of miles away in space, and this for very small obstacles, while for large ones the distance may be hundreds of millions of miles. That light waves do bend around obstacles is now very clear. Later scientists than Newton found that when light passed through very narrow slits a peculiar effect was produced, which is known as diffraction. Such a narrow slit should produce a similar image on a screen, but, as a matter of fact, it produces several, all of which except one are out of the line of sight, though they are faint. They can only be explained by the suppo-

sition that as the light passes through the slit the wave bends around the edges just as water does around the end of a pier.

Scientists having at last accepted the wave theory of light as correct, it was found that every known optical phenomenon could be readily explained by it. Furthermore, by various delicate devices the lengths of the light waves have been measured; which again would tend to prove the theory.

Not only are there water shadows and light shadows, but acoustic shadows as well. Let A in Fig. 33 be a sound proof wall,

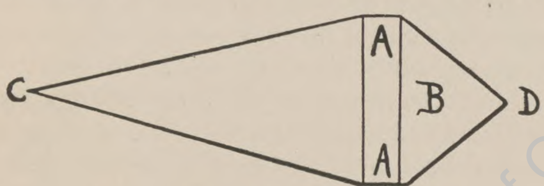

FIG. 33.

C the source of sound, and B the point where a listener is stationed. Here the sound will not be heard, but by moving back to D it will, since at this point the broken ends of the waves have come together. The space from A to D is a sound shadow.

In order that there may be waves there must also be a substance through which the waves may pass. It is known as the result of experiment that light passes with undiminished speed through a vacuum; the fact that it reaches us from the sun and heavenly bodies shows that it crosses space; hence it has become necessary for scientists to assume the existence of a peculiar, all-pervading matter, invisible and intangible, of so low specific gravity that it cannot be weighed, imcompressible as metal, yet so elastic that the minutest impulse received by it will be transmitted as freely as in jelly. There is no proof whatever that such

a substance exists, just as there is no direct proof of the truth of the wave theory itself; but judging from the results obtained and shown, the existence of both is assumed as the best explanation possible of all optical phenomena. This is why it is called a theory.

Waves of energy are of an infinite number of kinds; some coming to us from the sun, others produced here by artificial means. Each particular wave length produces its own effect, and all the waves taken together have been compared to notes of music and arranged in octaves. Of these a certain number of octaves contain all the light waves, but below and above them are longer and shorter waves, some producing heat, some electricity, some chemical effects, and other effects not yet well understood, such as the X-Ray and Radium rays.

The speed of light through the ether was first discovered by Roemer in 1676. Astronomers by many previous observations had established the fact that one of the moons of Jupiter made a complete revolution around the planet in 42 hrs. 28 min. 35 sec. From the data in hand Roemer had calculated out for a long time ahead just when this moon should begin to show itself from behind the planet. One evening, however, in looking for this moon, he noticed that it was about fifteen minutes late. This threw doubts on his calculations, but revision showed them to be correct, so that the loss of time had to be sought elsewhere. The reason for it became clear when it was seen that Jupiter was just then at the farthest possible point from the earth while Roemer's calculations had been made when the planet was nearest to the earth; hence the inevitable conclusion that the retardation of the appearance of the moon must be due to the increase in distance. On this basis the speed of light was found to be 186,000 miles per second, an enormous velocity, it is true, but still slow enough for

a distant star to be extinguished for millions of years before the earth's inhabitants would know of the catastrophe.

The speed of light has been verified by other methods of measurement, of which the following is one. Let Fig. 34 A be a large circular disc with holes drilled at small intervals near its periphery as shown. Behind one of the holes, X, let a telescope be placed at right angles to the surface of the disc; behind the next hole, Y, let a small but powerful light be placed. Now let the observer look with the telescope through the hole X at some convenient object, such as the side of a house, five miles away. He should have an assistant at this point, with whom he is in communication by telephone or otherwise, who will attach in the exact spot a mirror so arranged that the observer has it in the center of his telescopic field of view. This mirror is to be carefully adjusted so that he may see reflected in it the light which is beside his telescope. The rays of this light must first pass through the hole Y, then five miles away to the mirror, then back to the hole X, and finally through the telescope to his eye. This means nice adjustment, but it can be done. Now by a suitable mechanism the disc is made to revolve, under which circumstances the view of the light will at first be alternately cut on and off, but as the speed increases the light will at last disappear from view. This is due to the following cause: As the small holes come successively into position before the light, impulses of light pass across the five-mile space whence they return toward the telescope, but this they fail to enter because the periphery has moved sufficiently meantime so that the disc hole in front of the telescope has been carried so far to one side that the light fails to reach the instrument. A further increase of speed, however, will bring the light again into view, because each impulse will reach the mirror and return in the space of time required to move

the wheel one space. Under these circumstances if the light pass through X it will not reach the telescope through Y but through the next hole. A still further increase will once more cut off the view only to be regained when the speed becomes so great that the disc moves two spaces in one transit of the light and returns.

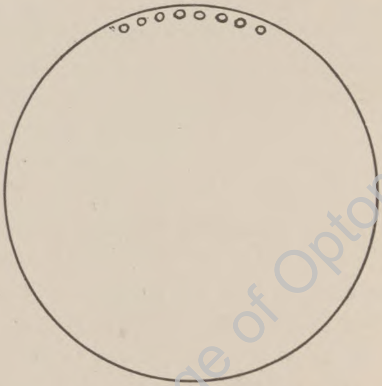

FIG. 34.

Knowing the circumference of the wheel, and the number of revolutions per minute the speed of the light wave may be calculated. The average of many trials gives about the same result as Roemer's calculations—186,000 miles per second.

QUESTIONS.

1. What is a shadow?
2. Name some varieties of shadows.
3. Illustrate with stone dropped in water.
4. Give a second definition of shadow.
5. Describe the old theory of light.
6. What is the speed of light?
7. What is meant by imponderable?
8. Why did Newton reject the wave theory of light?

9. Why can we not see around an obstacle?
10. What is diffraction?
11. Describe an acoustic shadow.
12. What is the ether, and its properties?
13. What proof is there of its existence?
14. Name some varieties of radiant energy.
15. What length waves produce vision?
16. Who discovered the speed of light?
17. How?
18. How long does it take light to come from the sun?
19. Describe the method of measuring the speed of light with a perforated disc and a distant mirror.

CHAPTER X.

The index of refraction of water is 1.33+ or one and one-third, which means that light passes through space one and one-third times as fast as it does through water. This is for pure water. The matter can be put the other way, if so desired, by saying that the sine of the incident angle is one and one-third times the sine of the refracted angle.

In crown glass, which is the kind in ordinary optical use, the index of refraction is about 1.5, though this is only approximate, since no two varieties will be found of exactly the same optical density. The index of refraction of a given dioptric medium is usually not so simple a formula as the above; but is very apt to be a long succession of figures, such as 1.52837629+, which will serve for an example. The range of the indices of refraction for glass may be as low as 1.4 or as high as 1.9, but usually it is 1.52.

It is to be noted that the index of refraction as given above is not for all the colors that make up white light, but is only the average. The extent to which the average index of refraction differs from the indices of the various colors, depends arbitrarily upon the specimen of glass used, though the order of the variation is usually the same. On the whole the difference between the indices of the different colors is not great, the violet having the greatest, the red the lowest and the yellow about the center place.

The fact that the speed of light is retarded when it meets glass

explains why a lens changes the direction of the rays; or this may be expressed in a better way by saying that the form of the light waves is altered, for, as stated before, light rays are merely a convenient expression to assist in the better understanding of optics. This change in the form of a light wave is shown graphically in Fig. 35, where A represents a point of light whence proceeds a hollow spherical wave shown in section at C D just before it reaches the lens B. It is evident that the center point of the wave has much more glass to traverse than the extremities, and that the amount to be traversed varies with the distance of any particular point of the wave front from the center outwards. Under these circumstances the wave after passing through the lens will have the form as shown in E H, so that all its particles as they advance will come closer and closer together until they meet in one monstrous wave upheaval at the focus F, whence they will advance spreading as they go with the form of the wave once more reversed.

The reason why the wave front C D is reversed will possibly be made more clear by noticing that the center of the wave must traverse the entire thickness of the lens, while at the extremities the amount to be traversed is much less. Now light moves through glass only at two-thirds of the speed through air; hence the ends of the waves will move faster than the center; hence the change in shape.

A practical though partial explanation of the change in a wave front may be made with a piece of soft wire bent as shown in C D. With this wire strike forcibly a column of artist's moist modeling clay of the shape of B. The resistance to the progress of the wire being the greatest at the center because of the greater diameter of the material, the wire will emerge with the direction of its curve changed.

Why does the wave front E H after passing F change its front? Why should not the wave commotion at F end the matter? To explain this point we must once more have recourse to a pond of still water. Suppose that we throw into such a pool two stones at different points, we will see that each one causes its own circular wave. The two will proceed outward in a circle until they meet. At this point there may result for an instant a wave of double size, or there may be a moment's calm, or there may be any size of wave between the two; it all depends upon how the

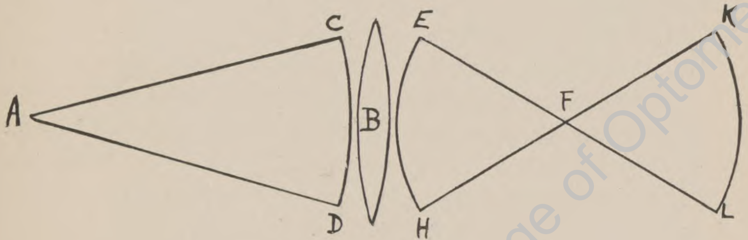

FIG. 35.

waves meet, crest to crest; trough to crest; or in some intermediate phase; if crest to crest, or trough to trough, there will be a double-sized wave; if crest to trough, one impulse will balance the other and there will be quiet; but no matter what occurs, each wave will cross its opponent and appear on the other side, to continue its way unaffected. The same occurs in the case of light. The same rule applies in the case of the individual particles of a single wave. In Fig 35 each portion of the wave front has its own direction which will remain unchanged unless it meets some optical obstacle. The wave front C D is affected by such an obstacle, the lens B; hence its form is changed to E H, but this

latter meets nothing in F, since there is nothing there, this being merely the focus where all the rays of E H meet in a point; hence each part of E H keeps its own path after meeting F as well as before. The reversed curve K L is the result.

There is a close relation between light and heat, and in regard to the latter it is sometimes said that there is no heat in space. If the sun gives out heat then the nearer we get to it the hotter it will be, yet the tops of high mountains are intensely cold at all times, and balloonists find the temperature of the air at a height of two or more miles almost unendurable because of its low temperature. Were a barrel of water fired from a powerful cannon to a height of ten or more miles it would probably freeze solid instantly. What the sun gives out is radiant energy, waves of infinite variety of length, and these becoming entangled in the vapor and dust of the air or in the soil and rock atoms of the surface are changed to certain other wave lengths by some mysterious process, which causes them to be perceived as heat. By analogy light is classed in the same way; that is, there is no light in space, only waves which we interpret as such. If we lose the organs that produce the sensation which we call vision then there is no more light for us. If the student feels skeptical and asks what do we find in space, the answer is waves, vibrating motion of the ether, radiant energy; a term that not only includes light and heat but many other phenomena of physical science.

QUESTIONS.

1. What do we mean when we say that the index of refraction of water is 1.33?
2. What is the approximate index of refraction of ordinary glass?
3. Give the minimum and maximum index of refraction of glass as a whole.
4. Which color of white light has the highest index of refraction?
5. Which the lowest?

6. Which is about medium?
7. Explain the change in form of a light wave produced by a plus lens.
8. Why should this change of form take place?
9. What occurs when two waves cross one another?
10. What occurs after the particles of a wave front meet in a focus?
11. What has the power of changing a wave front curve?
12. Give examples of the three previous questions.
13. Is there heat in space?
14. What is the reason?
15. Name some of the results of radiant energy.

CHAPTER XI.

The wave theory of light carries with it as a necessity the further theory that these waves have certain sizes, that is, they must measure a certain distance from crest to crest. By methods too difficult to be explained here these measurements have been found to be exceedingly minute, running from fourteen-millionths of an inch for the deep violet rays to thirty-two millionths for the deep red with the yellow about midway, twenty-two-millionths. This means that in one inch there are about seventy thousand extreme violet rays, about thirty thousand deep red rays and about forty-five thousand yellow rays.

In a soap bubble we see a most gorgeous display of colors, whose origin was long a mystery. The explanation on the basis of the wave theory of light is not at all difficult. White light, falling upon the exterior surface of the bubble is partly reflected, but for the most part passes into the substance of the bubble until again a portion is reflected from the second surface while the greater part passes on. The per cent. of light waves reflected from each of these surfaces is possibly five, but one reflection strikes the eye with just about the same force as the other. This means two light impulses of the same general character reaching the eye at the same time. It is the interference of these impulses or waves which under the right conditions produce the wonderful display of colors.

Every light wave, the same as all other waves, is made up of

alternate crests and troughs. If two similar crests coincide, the effect is doubled, that is to say, the brilliancy of the sensation of sight for this color is twice as much as the sensation produced by one wave alone. If the trough of one meets the crest of the other and are equal in intensity, or approximately so, then they neutralize each other. They do not destroy one another as is sometimes stated, because if they are to the slightest degree divergent in direction they will if no obstacle intervene sooner or later separate each to go its own way; both impulses are there; one in opposite phase from the other, so that the effect of one of the waves on the human eye is made null by the other; that is, the eye fails to see the color which corresponds. To produce such a result as this the two waves must be one-half wave length apart, as then crests of one and troughs of the other will fit in together. Where crest of one agrees with crest of the other the color will be intensified. Now for crest of one to correspond with trough of the other the thickness of the bubble must equal one-fourth of a wave length, since then the light reflected from the second or inner surface of the bubble must travel one-quarter of a wave length further to reach that surface and then return another quarter of a wave length, thus placing it on its return to the eye one-half of a wave length behind. If we call one wave X and the next Y, then X will first be reflected from the outer surface of the bubble to the eye, after which at the regular interval will come Y; meantime, the transmitted portion of X has gone on to the second surface one-fourth of a wave length away, where it is reflected just in time to meet and coincide with the trough between X and Y, a condition which results in the eye not seeing the color at all. Now let us suppose that the color thus obliterated is red, then the eye will see not white light but what is left of white light when the red is removed, and this will be green. Should the thickness of

the bubble be one-fourth of the length of the wave producing green, then the green will not be seen, but in its place will be that color which is left when green is taken from white. It is to be noted that as a bubble endures it grows thinner and thinner because of its growth in size without change in quantity of material, or to a slighter degree because of the evaporation from its surface, therefore the first color to appear will be that one which is complementary to the wave of greatest dimension, and since deep red light has the longest wave it will be the complementary color of this which will appear first green, while red is not apt to show at all because the bubble will need to be so thin that it bursts beforehand.

A complementary color is one that taken together with some given color will make white light. The following colors are complementary to each other:

Crimson, Moss Green.
Scarlet, Peacock Blue.
Orange, Turquoise.
Yellow, Blue.
Primrose, Violet.
Greenish Yellow, Purple.

To find what is complementary to a given color take a piece of paper of the color to be tested, place it in the center of a large piece of white paper; hold in the sunlight or other bright light, gaze at it steadfastly for half a minute; then turn the eyes to one side so that the white paper only is in line of view. There will appear an exact copy of the shape of the patch of color tested, but of the complementary color, very vivid and lasting for several minutes. The explanation is that the retina becomes tired for the color first gazed upon; then when the eyes are turned to the white background, that part of the retina which received the

image of the colored patch in the first place is lacking in sensitiveness for those rays from the white surface which correspond to the colors first seen and for which it is exhausted. Hence it only sees what is left; or, in other words, the complementary color. This physiological function is called retinal fatigue.

When we look at a rainbow it is sunlight we see by the way of certain raindrops. A certain portion of sunlight falls upon a mass of raindrops, which, as they fall out of the line of sight, are replaced by others, while the light shines on as steadily as before. This light reaches us by this particular route because the surfaces, internal and external, of the drops are properly placed so that the refractions and reflections which they undergo are just right to produce the result noticed. If we trace these rays that reach us in this indirect manner we will find that they first enter the raindrops to fall upon and be reflected twice from the inner rear surface, whence they pass out of the front of drops to reach the eye. It is true that there are other raindrops in other parts of the sky which reflect the sunlight to the eyes direct, while there are other drops so situated that they reach the eye after one internal reflection instead of two, but in both these two cases the amount reflected is not sufficient to be recognizable while those which are described above, those which cause the rainbow to appear, do send enough sunlight to the eyes to make us see it clearly. For this light to reach our eyes the drop must be in the right place, or, what amounts to the same thing, any drop falling from that place must leave several in the same line of sight behind, which is exactly what happens with falling rain; many raindrops are always in the line, though not always the same ones.

All of the rays of lights which follow the path above stated, from sun to drops, thence to eye, suffer dispersion because the

refraction they undergo in their passage through the drops is not the same for all the wave lengths; or as generally stated not the same for all of the colors, and if we will divide the patch of sky containing the raindrops into narrow bands one of these bands will be at the right point so that the eye sees yellow light, therefore the sky at that point seems yellow. From another band will come red light; from another violet light, with connecting colors and shades of color between. All these colors which we see seem to form a bow because for us to see the sun by this path the bands of raindrops producing the refraction and dispersion must be in that form.

If we draw a line from the eye to any portion of the red curved line in the rainbow and thence to the sun the angle formed will be forty-two and one-half degrees. For the violet it will be forty and one-half. Hence the total width of the rainbow is two degrees. The moon is three degrees wide, and this will serve as a comparison.

The question is sometimes asked, Has the rainbow a shadow and where should we look for it? The rainbow is an optical illusion. The refraction and internal reflections of a multitude of raindrops two degrees wide break up sunlight so that on the retina of the observer's eye the bow is formed, and thence it is optically projected outward to the sky.

QUESTIONS.

1. Between what two points must waves be measured to get their size?
2. How long are violet, red and yellow light waves?
3. How many surfaces of a soap bubble reflect light to the eye?
4. What portions of light waves are reflected from the surface of the soap bubble?
5. If two light waves of similar size meet, crest to crest, what is the result?

6. If the crest of one coincides with the trough of the other, what results?
7. Do waves of light ever destroy one another?
8. How thick must the film of a soap bubble be so that some particular color may seem to be destroyed?
9. What other color is seen in this case?
10. Which colors are apt to appear first on a soap bubble, and why?
11. State some pairs of colors which are complementary.
12. How can a complementary color be discovered?
13. What is retinal fatigue?
14. What are we really seeing when we look at a rainbow?
15. Why does a rainbow seem stationary?
16. Describe the course of a ray of sunlight in reaching the eye when we look at a rainbow?
17. Where is the rainbow? In the sky?
18. Does a rainbow change its place when we do?
19. Do two people look at the same rainbow?
20. What is the reason that we see rainbows?
21. What is dispersion?
22. Why is a rainbow arched?
23. What is the width of the rainbow?
24. Of what angle are the red rays the apex?
25. Of what angle are the violet rays the apex?
26. How many degrees wide is the moon?
27 How would you figure this out?
28. Has the rainbow a shadow?
29. What reason is there for calling the rainbow an optical illusion?

CHAPTER XII.

MINUS LENSES.

A minus or negative lens is one which causes rays of light reaching it from a distance to diverge as though they came from a point near it and on the same side as the object called its focus.

Minus lenses may have three forms—bi-concave, plano-concave and minus periscopic. (See Fig. 36.)

FIG. 36.

There is one point where minus lenses can be readily distinguished from plus ones. This is in the thickness of their edges and the thinness of their centers. The thinness of the centers is an optical advantage in lenses of high power, since thickness of glass always causes a certain amount of aberration which is avoided in minus lenses simply because the centers are thin.

The extent to which a minus lens will cause rays of light to diverge depends on the dioptric power of the glass and upon the curvature of the surfaces. The glass used in the practice of

optometry being all of about the same optical density, the curvature is the main consideration.

The relation of the curvature of minus lenses to the dioptric power of these surfaces is the same as is the case with plus lenses, namely, the radius of the curvature expressed in dioptres will be twice the dioptric power of the individual surfaces.

Let A, Fig. 37, be a minus lens, of which one surface is struck on a curvature of ten inches while the opposite curve is on a curvature of twenty inches. Then the dioptric power of the first surface will be two, while that of the second surface will be one,

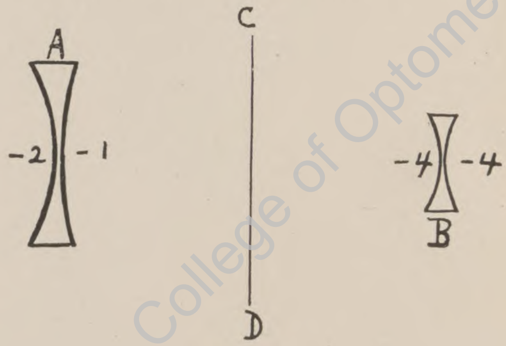

FIG. 37.

or a total of minus three dioptres. In the same figure let B be a bi-concave lens with each surface on a curvature of five inches, then the dioptric power of each surface will be four dioptres, or a total of minus eight dioptres.

It will be seen that the sharper the curve the greater the power of the lens. In Fig. 37 B has a sharper curve than either curve of A because it deviates more from the direction of the straight line C D.

The power of a minus lens is the algebraic sum of the dioptric powers of its two surfaces. In Fig. 38 the three lenses shown all have the same dioptric power, since the sums of their two curvatures are equal.

Since conjugate foci are any two points where image and object may be interchanged and still the former be at a focal point; therefore minus lenses cannot have conjugate foci, since no such interchange can take place, and in fact a minus lens does not form an image. It does diverge rays of light, which if traced in a reverse direction will appear to come to a point, and this point is called a virtual focus.

FIG. 38. *

A virtual focus may be defined as the location of an imaginary point where an image would be found if the direction of the rays of light were reversed. It is also sometimes called a negative focus.

Fig. 39 shows a virtual focus. Let the parallel rays of light, supposed to come from a distant illuminated object, meet the lens B where divergence of rays takes place, and the rays of light take such a direction that they seem to come from A. It is to this point that the name virtual image is given. In this particular case, the rays being parallel, the point A is called the principal focus of the lens. Its distance from the lens expressed in dioptres is for ordinary glass equal to the dioptric value of the two surfaces of B added together.

*NOTE.—In the third lens the outside surface is intended to be + 1 instead of — 1.

In Fig. 39 the direction of the light rays is from C to lens B and then to D, and from F to lens B and then to H. The rays only seem to come from A.

Suppose now that the object which is the source of light be brought nearer so that it is at A in Fig. 40, which is a duplicate of Fig. 39. Then the same thing will happen again. The rays of light will diverge, this being the effect of minus lenses, and the virtual image will be found still nearer to the lens at F. It is therefore evident that A and the distant object

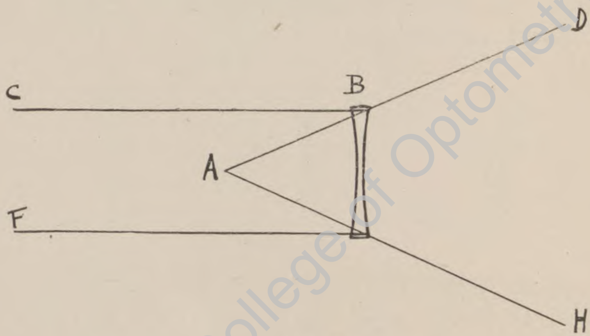

FIG. 39.

of Fig. 39 were not conjugate and that with minus lenses no matter how far or near the object may be the image is always on the same side of the lens and nearer to it, and that this image is always a virtual one. The nearer the object to the minus lens the nearer must the virtual image be.

There is no aerial image at the focus of a minus lens, and this can be proved by trying to catch an image at that point with a white screen, which only results in cutting the light off from the lens.

The relation of size of object and of image is the same, as is the case with plus lenses; that is, the size of one is to the size of the other as the distance of one from the lens is to the distance of the other from the lens. For example, if the object is twenty feet distant and the image is found at eight inches, then the diameter of the object will be thirty times that of the image. The location of the virtual focus of a minus lens may be plotted out in a manner similar to that employed in finding the conjugate

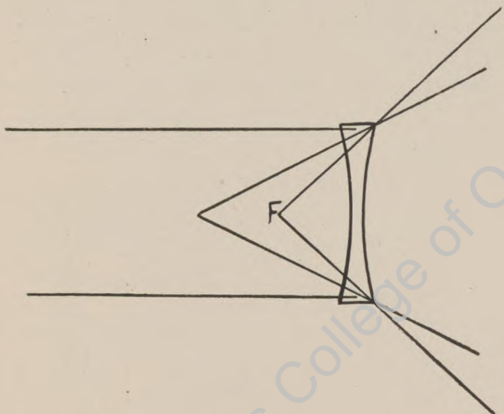

FIG. 40.

focus of a plus lens. From some point of the object not on the principal axis of the lens lay off a straight line which will pass through the optical center of the minus lens. This will be a secondary axis and will suffer no refraction; also on some point of this axis all the other rays emanating from the point of the object selected will meet, such meeting point being the virtual focus. Since all the rays from the selected point in the object

will meet in this focus, therefore if we can find the point where one of these rays crosses the secondary axis we will also find the point where they all cross, which is the location of the virtual image sought. To this end lay off a line from the selected point of the object parallel to the principal axis of the lens until it meets the lens; here it will be diverged so that its new ray will appear to come from a point at the principal virtual focal point of the given lens. Trace this backward until it intersects the first line drawn, and this point of intersection will mark the plane of the required virtual focus. This method is shown in Fig. 41, where R is the minus lens, O is the object, M is the principal virtual focus, the location of which depends on the

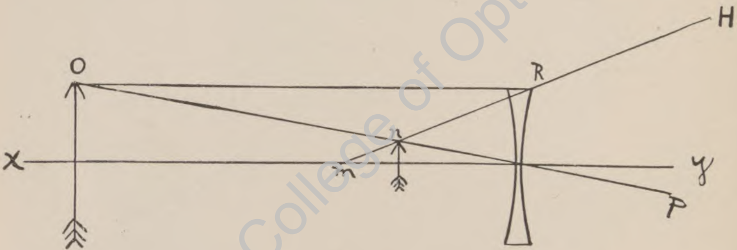

FIG. 41.

dioptric power of the lens. From some point in O, such as O, draw the secondary axis O P, which, of course, will cross the optical center of the lens R. Next draw the line O R parallel to the principal axis X Y until it reaches the lens. Here it must diverge so that when traced backward it will seem to come from the principal virtual focal point M. Where these two lines cross will be the location of the desired focus. By taking any number of other points of the object O and proceeding in the same man-

ner it will be found that the image is an exact duplicate of the object at the point N.

The size of the virtual image will depend upon its relative distances from the center of the lens. If we assume that the distance from O to R is thirty inches and the distance from N to R is five inches and that the diameter of O is one inch, then the diameter of N will be one-sixth of an inch.

The location of a virtual focus of a minus lens may be calculated in the following manner: Measure the distance from lens center to object in inches and convert same into dioptres

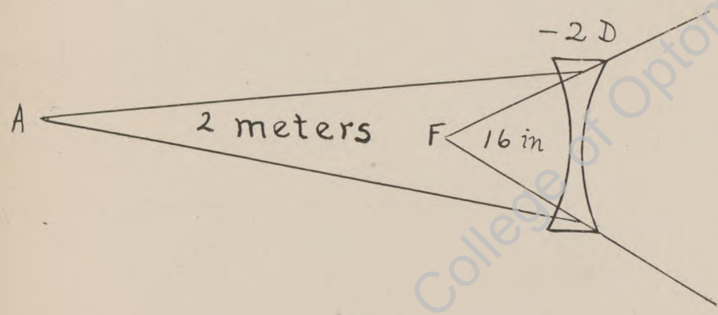

FIG. 42.

of distance. To this value add the dioptric strength of the lens. This will be the distance of the virtual focus expressed in dioptres. Convert this value into inches for the required answer. In Fig. 42 it is plain that the rays of light proceeding from the point A have a certain amount of divergence, which will be increased because a minus lens is a diverging lens, therefore the total dioptric divergence of rays from F will be the sum of the radial divergence of the rays from the point A plus the minus

dioptric power of the lens used. Hence the two values must be added.

In Fig. 42 let A be an object at a distance of two meters from the minus 2 dioptre lens. Since a distance of two meters is the same as one-half of a dioptre, then to find the focal point F we must add this value to the dioptric value of the lens, which will make a total of two and one-half dioptres. Change this to distance inches and the result is sixteen inches for the distance of the focus F.

QUESTIONS.

1. What is the effect of a minus lens on parallel rays of light?
2. Name the three forms of minus lenses.
3. Which have the thinnest centers, plus or minus lenses, and why?
4. Why are minus lenses optically better than plus lenses?
5. Name the two factors in minus lenses which cause varying degrees of divergence.
6. What is the relation of curvature to dioptric value in minus lenses?
7. Give some examples.
8. Which size of circle, that of one inch diameter, or that of two inch diameter, has the greatest curvature, and why?
9. What is meant by the term "algebraic sum"?
10. What is a virtual focus?
11. What is a negative focus?
12. What is the difference between dioptric lens value and dioptres of distances?
13. On what rule does the size of a virtual image depend?
14. How may the virtual image of a minus lens be plotted out?
15. Explain why this is a correct method.
16. How may a virtual image of a minus lens be calculated?
17. Give some examples.
18. Suppose the distance of the object from the lens is given; also the distance of the image; how can the power of the minus lens be calculated?

CHAPTER XIII.

When a minus lens is held a short distance in front of the eye and moved from side to side distant objects seen through the glass will appear to move in the same direction as the lens is moved. This is an optical illusion of which we make use in testing lenses. The explanation of the apparent movement is this: When the lens is moved its virtual image also moves and in the same direction, but since this image is formed on the same side of the lens as the object while the eye is on the opposite side, therefore this image, which is visible to the eye, produces exactly the same effect on the retina as would be the case if it had an actual existence. The rays of light apparently coming from this virtual focus reach the eye and are focused by it on the retina, but in accordance with the optical principles; if the object is to one side of the principal axis of the lens its image will be on the other; hence if the location of this virtual focus changes from one side to the other of this axis the retinal image will go the opposite way, thus a contrary retinal transit in every case; but in accordance with the laws of the sense of sight any movement across the retina corresponds to an opposite movement of the source of light which causes it; hence if by artificial means the image is made to move when the object does not, then the eye be deceived and will see the object move in the contrary direction. Since the movement of a minus lens causes the retinal image to move in the contrary direction to that of the lens and since this movement is interpreted by the brain centers as

a motion of the object in the reverse direction, therefore the distant objects as seen through the minus lens will appear to move in the same direction as the lens is moved. Let Fig. 43 show a minus lens in two positions, let B be a distant object, and let C be a strong lens corresponding to the eye. It will be noted that when the minus lens is in the upper position the retinal image will be at X while with the minus lens in the lower position the retinal image is at Y; that is, it has moved up while the lens has been moved down. This means in accordance with the laws of physiological optics that B seems to move down also; that is, in the same direction as the lens is moved.

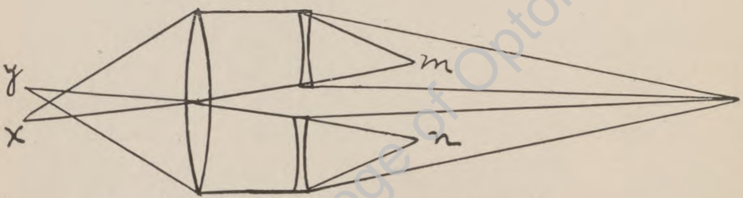

FIG. 43.

Another explanation is that the movement of the lens down (Fig. 43) moves the virtual focus down, so that in the first case the object seems at M and in the second case at N. Let Fig. 44 show these two points without the minus lenses in place. It will be seen that their respective retinal images are reversed and that movement from M to N outside the eye corresponds with retinal movement from R to S within the eye.

If a minus lens is held a short distance in front of the eye and then moved forward and back distant objects seen through the lens will seem to change in size, increasing as the lens is drawn near and decreasing as it is pushed away. Since it is evident

that the object does not change there must be a change in the size of the retinal image which deceives the brain centers whose function it is to exercise and control the sense of sight. Supposing the minus lens to be held at a certain distance from the eye there will be formed between the lens and the object a virtual image which will have a certain size depending on the distance of the object and the dioptric power of the lens. The rays

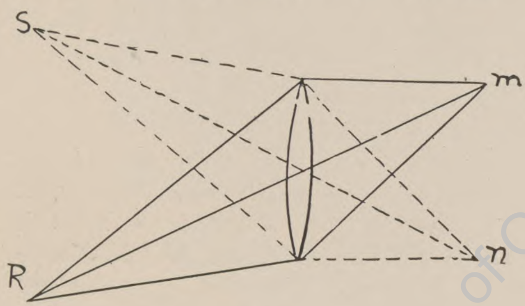

FIG. 44.

coming apparently from this virtual image will be focused by the dioptric ocular media upon the retina, and will also have a given size, small, of course, but dependent upon the distance of the virtual image. Now push the lens away a few inches. The virtual image will increase a little in size theoretically, but the relative distance is so little changed that this increase of size will not be noticeable and we may call it practically unchanged. But this is not the case with the retinal image, since the distance of the virtual image in the first place was short, hence the addition of the extra two or three inches means a considerable decrease in size, which is very easily comprehended. Now when a distant object actually grows in size the retinal image will

also increase, and since the brain always interprets decrease of size of retinal image as a decrease in size of the object, therefore when we decrease the size of this image we ascribe the decrease to the apparent cause. When the glass is drawn toward the eye the contrary effect is produced, and as a result of the illusion the object seems to increase in size.

In Fig. 45 let rays of light almost parallel coming from a distant object fall on B a minus lens and thence through a strong plus lens corresponding to the eye. The parallel rays of light will have for a virtual focus the point F in both cases so nearly the same distance from B that they may be considered as of equal size, though there is an extremely slight difference, but this is not the case in reference to the distance of F from the strong lens, the distance in the upper drawing being much less than in the lower. Now as the image of an object is in direct proportion to its distance from the lens in comparison with the distance of the image from the lens, therefore it will be seen by the figure that since F is alike in size in both cases the image in the upper drawing must be of greater size than is the case in the lower one; hence the distant object will seem to decrease in size as the minus lens is withdrawn.

The dioptric power of a minus lens may be measured in several ways.

First—By means of a lens measure. Apply the contact points of this instrument first to one side of the lens and then to the other. The algebraic sum of the two readings will be the dioptric value of the lens required.

Second—By neutralizing by means of lenses from the test case. Take the minus lens to be tested, hold it in front of the eye and look at some distant object. Move the lens from side to side and from front to rear, and estimate its dioptric power.

Take a plus lens from the trial case of this estimated strength, hold the two together and try again to see whether all apparent motion or growth of the distant object has ceased. If not take a stronger plus lens and repeat the test. Should the direction of motion in relation to the movement of the lens be reversed then take another plus lens of less power, and so continue until the two glasses used together act as a plano glass; that is to say, un-

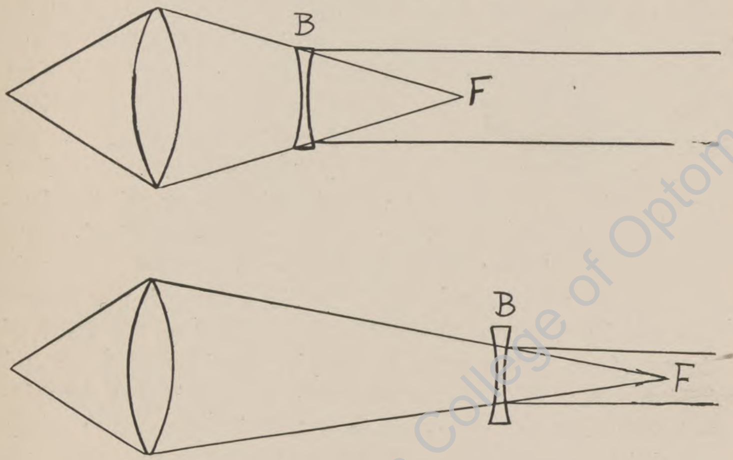

Fig. 45.

til distant objects seem stationary no matter what motion is given the two glasses together. The plus lens which produces this result is the measure of the power required but of the opposite sign. Very close and quick work can be done in this way.

Third—Where both surfaces are concave the following method by reflection can be used: Hold the lens so that a distant bright light may fall upon its surface. Between the light and lens, but

a trifle to one side, hold a white card. Move this latter about until an image of the distant light is made upon the card by reflection. This image though comparatively faint is easily visible, and is formed by the concave surface of the lens acting as a mirror. Possibly not more than 5 per cent. of the light received by the lens is reflected in this way, the balance being refracted; but this 5 per cent. is sufficient for the purpose. Measure the distance of this image from the surface of the lens, express the distance in dioptres and divide by four; the result will be the dioptric value of the first surface of the lens. Test the opposite side in the same manner and add the two results. For example, suppose one surface forms a clear image of the distant source of light at ten inches. This corresponds to four dioptres; divide by four and the dioptric power of the lens surface is found to be one dioptre. If the other surface forms a clear image four inches from the lens surface, then since this corresponds to ten dioptres the dioptric value of the second surface will be two and one-half, which, added to the value of the first surface, makes a total value for the lens of three and one-half dioptres. The reason we divide by four is that the relation of the radius of curvature in a lens is as 2 to 1; while the catoptric relation of the curvature of a concave mirror surface to its focal point is as 1 to 2; from which we get the relation of 4 to 1 for concave mirror focus and dioptric power of a similar concave surface.

Fourth—By the illuminated area method. Hold a minus lens so that the light from a small, bright, distant light may fall upon its surface. Place behind this a piece of white card on which is drawn a circle or oval to match the contour of the lens, but with twice the diameter in every corresponding direction. This arrangement will cause a bright area to appear on the card, which,

when the card is held close to the lens, will be of the same size, but which will increase in diameter as the lens is drawn away. When this increase in size has continued until the area enclosed by the lines on the card is entirely filled with light then measure the distance the two are separated, and this will be the focal distance of the minus lens. The reason why this is true will be made plain by Fig. 46, where the parallel lines are to represent the light coming from the distant object. A represents the lens and C the screen on which the light area is to be shown. As

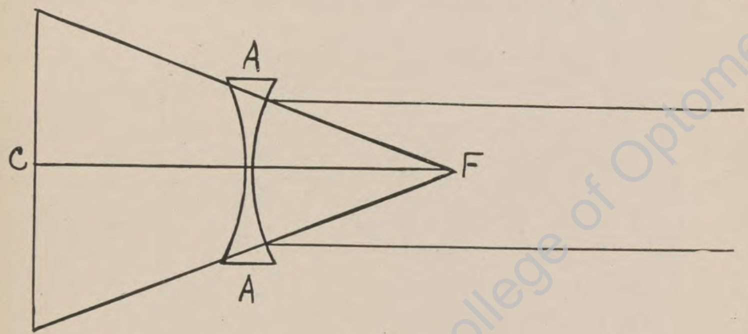

FIG. 46.

the result of the dioptric power of the minus lens the parallel rays shown will be refracted so that they seem to come from the focal point F, which will produce the same result as though they really did come from F. From this it follows in accordance with geometrical rules that if the distance from A to C is the same as from A to F the size of the light arear will be double the size the lens A. Conversely, if C is double the area of A the distances A F and A C must be equal, and being equal A C will be exactly the same distance as the focal distance of

the lens. Express this distance in dioptres and the result will be the dioptric of the lens under test.

Fifth—By over-correction. Hold against the minus lens to be tested a plus lens of higher power. Get the focal point of the two together in accordance with any of the focal methods employed for plus lens. Then find the value of the plus lens used alone. The difference between this value and that of the two lenses taken together will be the dioptric power of the minus lens sought. For example, if the two lenses together have a power of one dioptre and the plus lens alone has four dioptres,

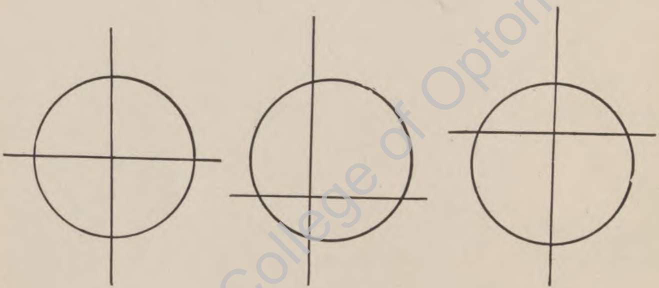

FIG. 47.

then the value of the minus lens is three dioptres.

In order to find the optical center of a minus lens proceed in the same way as with plus lenses. Make a cross of two long lines drawn at right angles. Hold the lens to be tested in front of the intersection of these two lines, and by trial bring the lens to such a point that when the lines are looked at both through the lens and outside they will appear unbroken. When this result is reached mark with an ink dot the place where the lines seem to cross on the face of the lens. This spot, if the lens is

accurately centered, will also mark the geometrical center; if it does not the lens is decentered either purposely for prism effect or as the result of poor workmanship, usually the latter. Fig. 47 shows such cross lines appearing unbroken; in one case the lens properly centered, in the two others not.

QUESTIONS.

1. How do distant objects seem to move when seen through a minus lens moved from side to side?
2. What is the reason for this apparent motion?
3. What is the relation of the location of retinal images to the objects that cause them?
4. In what way does the brain interpret motion of retinal images?
5. If a minus lens be held in front of the eye and then moved back and forth, what will be the apparent action of distant objects seen through the lens?
6. What is the reason for this?
7. Under what circumstances is a virtual image visible?
8. What is the relation of size of retinal image to size of object?
9. How is the power of a minus lens measured with a lens measure?
10. How do you proceed to neutralize a minus glass with the lenses from the test case?
11. Explain the method of measuring bi-convex lenses by reflections from their surfaces.
12. How much light is reflected from the surface of unsilvered glass?
13. What relation does the curvature of a concave mirror bear to the distance of its focus?
14. Describe the method of finding the dioptric power of a minus lens by the size of the illuminated area it causes.
15. Give the principle involved in this method.
16. How may a minus lens be tested by the method of over correction?
17. Give an example.
18. How can the optical center of a lens be found?
19. What is the geometrical center of a lens?
20. What is a decentered lens?
21. What is the usual cause of a decentered lens?

CHAPTER XIV.

THE ACTION OF COMPOUND LENSES.

A compound lens is one made up of two or more simple lenses, and the dioptric power of the combination will vary with the dioptric value of the respective lenses and their distance apart. Such a combination may be made up of all plus lenses, of all minus lenses or of a mixture of both kinds. If two plus lenses of equal power be taken and placed close together their combined power will be a trifle less than the sum of their powers, but the decrease will be so small that it may be disregarded; that is to say, each lens has exerted its full refractory power upon the wave passing through it. Separate the lenses now until the image made by the first one falls exactly in the center of the substance of the second one; now the first lens exerts all its dioptric power while the second exerts none at all, since the image being between the two surfaces the effects of one counteract the effects of the other.

If, for example, we take two lenses each of four dioptres, the value of the two when close together will be eight dioptres, while when separated sufficiently their power will simply be the power of one, namely, four dioptres, while at intermediate points the power of the couplet would vary inversely as the distance apart of the two lenses. Some instruments for testing ocular refraction are based on this property of an adjustable lens couplet. Fig. 48 shows such a cuplet with the lenses in three positions—

close together, fully separated, and midway between the two. It will be noted that as the lens B is withdrawn the image moves in the same direction though not so rapidly.

It does not matter what the relative strength of the lenses may be, whether of equal power or not the general result is the same.

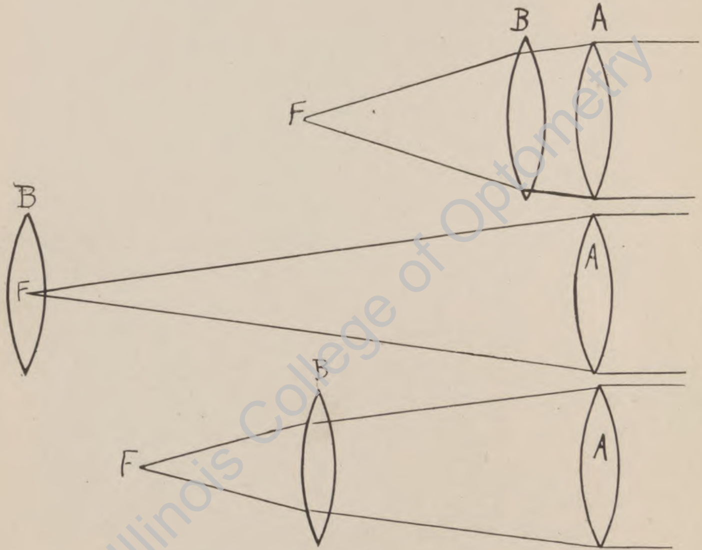

FIG. 48.

If the couplet be still further separated the image of one of the lenses may serve as the object of the other with the result that a second image will be formed which will be erect instead of inverted; that is to say, the first lens produces an inverted image of the object and the second image inverts the inverted

image and thus renders its directions the same as the original object. It is on this principle that terrestrial telescopes are made. For astronomical telescopes it makes no difference whether the stars and planets are seen inverted or not, while the light absorbed by the extra lens interferes somewhat with the efficiency of the instrument.

If the lens couplet is made of a minus lens and a plus lens some unexpected results will become manifest, for if the two

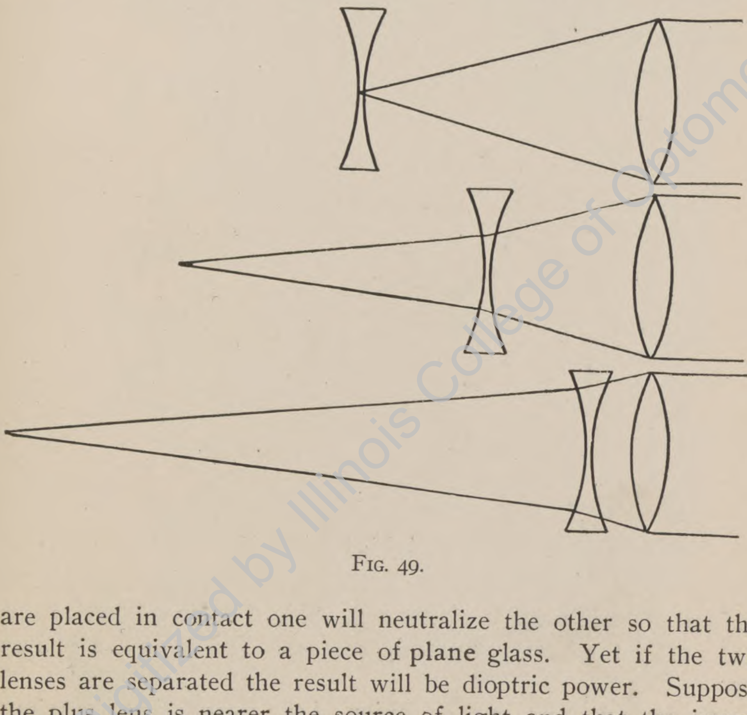

FIG. 49.

are placed in contact one will neutralize the other so that the result is equivalent to a piece of plane glass. Yet if the two lenses are separated the result will be dioptric power. Suppose the plus lens is nearer the source of light and that the image

formed by it is allowed to fall exactly upon the center of the other lens, then this second lens will produce no effect, since one surface is to the front of the image and the other is to the rear, one thus counteracting the effects of the other. The dioptric power of the combination therefore will be identical with that of the plus lens alone. Now as the minus lens is brought closer and closer it will counteract more and more the converg-

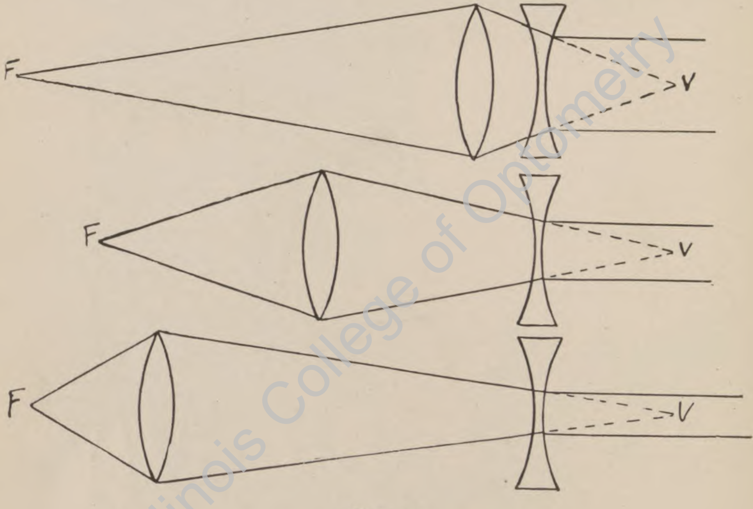

FIG. 50.

NOTE.—The lens may be carried indefinitely in the direction of F.

ing effect of the plus lens until one exactly offsets the other, so that all dioptric value is obliterated. It will be noted that as this takes place the image constantly recedes. This is shown in Fig. 49.

Suppose we reverse the position of the lenses. Let the minus

lens be nearest to the object, then this lens will tend to form a virtual focus between the object and itself. This tendency when the two lenses are in contact will be exactly counterbalanced when the two lenses are in contact and the result will be no dioptric power at all. When they are separated, however, the combination will act similar to a simple plus lens. The location

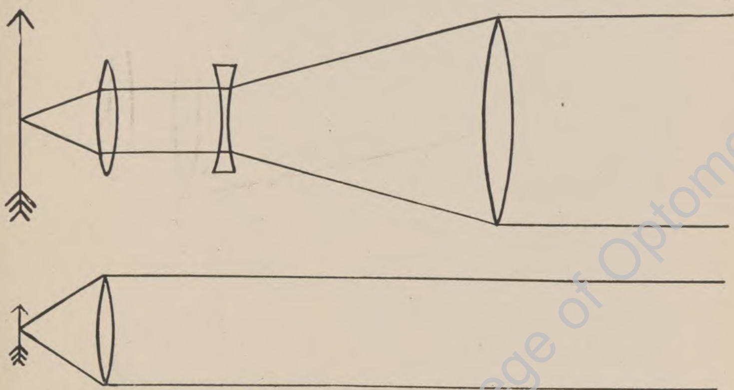

FIG. 51.

NOTE.—The narrowness of the page makes it impossible for the various lenses above shown to be of the right size and focus, but the principle involved is the same.

of the virtual focus of the first lens is also, when the two lenses have equal dioptric values, the principal focus of the plus lens. Now when the location of an object is farther from a plus lens than its principal focus it will form a conjugate image somewhere. Where the distance of such object is little in excess of the principal focal length the image will be at a great distance and of enormous size, but as the distance of the lens from the object increases the image both steadily draws nearer and

becomes smaller. From this it follows that when the minus glass is toward the object an image will be formed from the plus lens that follows it, no matter how far that plus lens may be withdrawn. This is made clear in Fig. 50.

Opera glasses are constructed on the principle that a couplet made up of a plus and minus glass has plus dioptric power. In

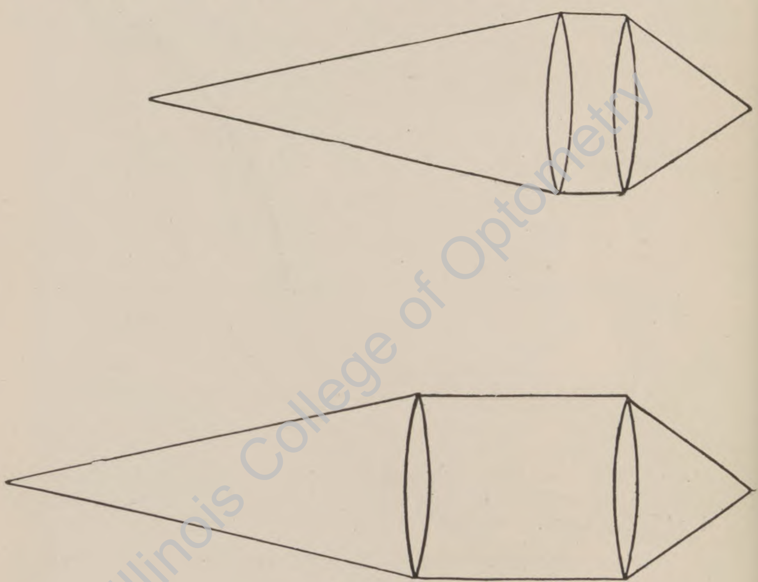

FIG. 52.

Fig. 51 shows the objective of an opera glass, usually achromatic, and the minus eye-piece, which is a minus lens of high power. The rays of light from the object pass through the plus lens as shown. They would reach their focal point but the minus lens intervenes. This later causes the rays to diverge until

they are parallel, when they enter the eye represented by the small plus lens to be focused on the retina, producing a larger image than would be the case were the opera glass not used, as shown in the lower drawing.

One practical use to which a double lens is put is for stereopticons or magic lanterns. In this instrument a bright light is concentrated on some transparent object and then an enlarged picture of this is formed on a white screen at a convenient distance. In Fig. 52 suppose a brightly illuminated object near to the double plus glass. With these glasses close together the image will be nearer and smaller when the lenses are near together, and vice versa. It is evident that with the instrument in place a clear image of the object can be formed at any point across the room by simply adjusting the distance apart of the two lenses until the details come clear.

QUESTIONS.

1. What is a compound lens?
2. How does distance affect the power of a plus couplet?
3. Give some examples.
4. What form has an image which is the result of two lenses so far separated that the image of one becomes the object of the other?
5. What is the difference between an astronomical and a terrestrial telescope?
6. Will a minus lens always exactly neutralize a plus lens of the same dioptric power?
7. Under what circumstances does one of the lenses of a couplet produce no dioptric effect?
8. In a couplet composed of a minus lens and a pus lens, suppose the minus lens nearest the object, at what distance will the plus lens cease to form an image?
9. Describe the internal arrangement of an opera glass.
10. How is the lens of a stereopticon or magic lantern adjusted for a given distance?
11. Why does an opera glass make an object look larger?

CHAPTER XV.

SOME MEASUREMENTS.

To find the magnifying power of a telescope or opera glass look through the instrument at bricks in the side of a house about thirty feet or so away, at the same time keep the other eye open. The eye looking through the instrument will have formed upon its retina an image of a certain size. The size of the image on the retina of the other eye will be much less. Now compare the two images as they seem to appear. If the width of one brick as seen in the telescope occupies the space of ten bricks as seen by the naked eye then the instrument has a magnifying power of ten diameters. Sometimes it is not easy to see both images at one time, first one eye seeing clearly and then the other, but all that is needed is practice to become expert in the test.

To find the magnifying power of a microscope. Take a rule divided into small divisions and place upon the stage of the instrument. Focus the tube till the rule is seen clearly. Place on the table by the side of the instrument and ten inches from the eye another similar rule. Look at this with one eye while the other eye is looking at the focused rule on the stage of the microscope; with a little practice both rules will be seen. Now compare one of the spaces on the rule in the instrument with the width it seems to cover on the one lying on the table ten

inches from the eye. If one division of the scale, say one-thirty-second inch, in the microscope seems to cover two inches of the one on the table then the magnifying power will be sixty-four diameters. If a less distance than ten inches is used the

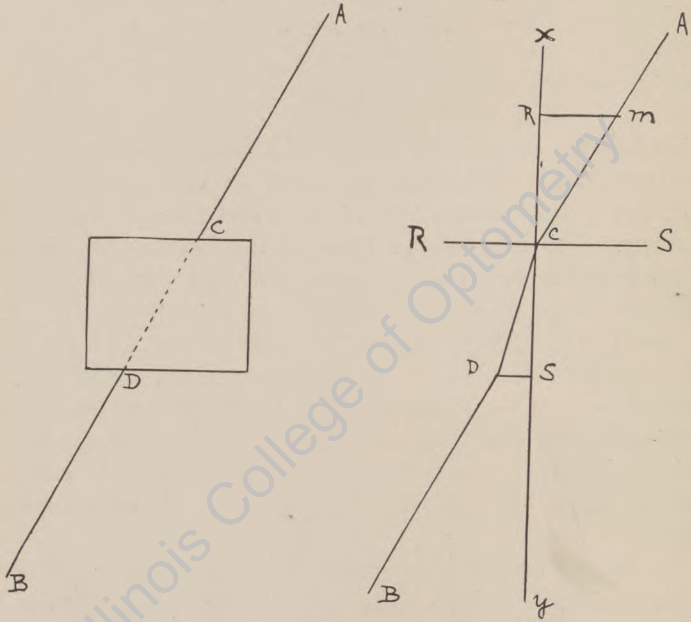

FIG. 53.

magnifying power will appear to be less; if a greater distance is used it will seem to be more, but ten inches is the standard in common use.

To find the index of refraction of any solid dioptric medium. Take a thick block of glass as shown in Fig. 53; place it on

a sheet of paper in the middle of a table. At the points A and B about six inches from the two opposite sides of the block stick two pins, so placed that a line joining them will cross the block obliquely. Sight the eye along this imaginary line, and at C and D close to the block of glass place the pins so that all four may be in one straight line. Be careful that all these are vertical. Now remove the glass, and it will be seen that the four pins are not in a straight line. Draw the lines A C and B D, also join C and D. Replace the block in position, and the line (the pins removed) will seem straight again, showing clearly that in looking from B to A the direction of the light has changed twice, and this is due to the refractive power of the block. Trace the outline of the glass block and remove again. At the point C draw the line X Y perpendicular to the surface R S. The line A C is evidently the incident ray, and the line C D the refracted ray. At the point R draw the sine of the incident angle, and at the point S, at the same distance from C as R, draw the sine of the angle of refraction. Measure these two sines carefully and compare. Divide the length of the sine of the refracted angle P S into the length of the sine of the incident angle R M and the quotient will be the index of refraction required.

Another and more exact method. Let A, Fig. 54, be a piece of the glass to be tested cut in the form of a right angled prism, as shown. Let B be a small bright light which is to pass through pin-holes in discs as shown adjusted so the holes may be exactly on the same level; let R F be the portion of the arc of a circle made of metal or other material and divided off into degrees, commencing at the point O perpendicular to the face of the prism at X and described on X as a center. The glass A must be placed with the vertical side on the side toward the source

of light, so that this may pass the first surface without being refracted. Let the angle of the prism, or wedge, A be any convenient value such as thirty degrees. Under this arrangement a very narrow beam of light from B will pass through C and D, meeting the first glass surface vertically, as stated above, and passing through the substance of the prism without refraction until it meets the second surface at the point X, where it will be refracted in the direction X R. This ray meets the second surface at the same angle to the perpendicular O X as the angle of the prism (stated above as thirty degrees). After emerging from the glass it forms with the perpendicular a new angle shown on the arc of a circle by the number of degrees between O and R. Consult a table of natural

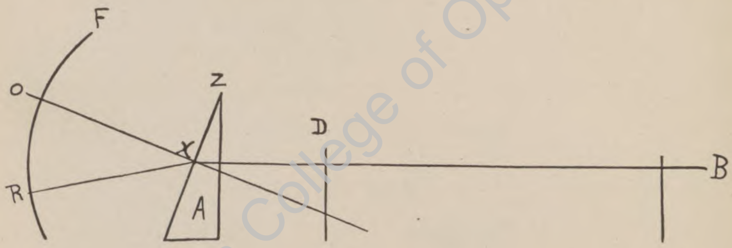

FIG. 54.

sines, get the sine of thirty degrees from this table, also the sine of the angle O X R, divide the first into the second and the quotient will be the required index of refraction. The course of the rays may be rendered visible by the use of smoke.

To measure out the focus of a lens of a given index of refraction. For the sake of simplicity of calculation let the lens take the form of a plano-convex glass and have the ray of light to be used as the basis of the calculation meet the plane surface

at right angles. This reduces the refraction to one surface. Let D, Fig. 55, be the plano-convex lens with the center of curvature of its curved surface at B. Draw the principal axis P V, also C D, parallel to this axis. From B, the center of curvature, draw B F, intersecting C D at D; then D F is a perpendicular to the curved surface at D; extend C D to L. Let the index of refraction be 2., then C D will be refracted on passing out of the lens so that the angle A will be doubled; but by geometry angle A equals angle B, also by the refractive power of the curved surface angle A equals C. Again, by geometry angle C equals D; therefore H, the principal focal point of the lens A, and the radius of curvature are equal; whence

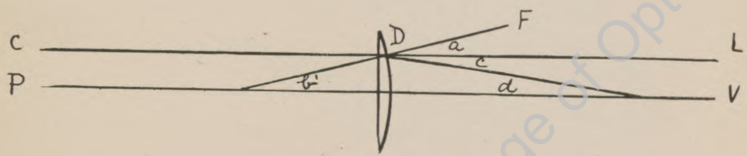

FIG. 55.

we might make the rule that for lenses whose index of refraction is 2. the dioptres of curvature and the dioptres of power are equal. Now in the case of lenses made of ordinary glass the dioptres of curvature are twice the dioptres of power. Hence if two lenses of the same curvature be taken, one of 1.5 index of refraction and the other of 2., the latter will be twice as strong as the former. With an index of refraction of 3. the rule may be worked out in the same way that for lenses having an index of refraction of 3. the dioptres of curvature are one-half the dioptres of power, whence with two lenses of equal curvature one of common glass with index of ref. of 1.5 and the other of

some other substance with an index of refraction of 3., the latter will be four times as powerful as the former. From all of the above we can assume a general rule for all indices of refraction as follows: From the index of subtraction subtract unity. The remainder will show the relation of the dioptres of power of any surface to the dioptres of the radius of the curve on which it is struck. For the index 1.5 this would be .5; for the index of 2, it will be 1.; for the index 3.5 it will be 2.5, etc.

The above is for small areas only and at the center of the lens, which is the usual custom in calculating and making all lenses. It is practically right for all ordinary sizes of lenses, and, in fact, there is no other rule.

QUESTIONS.

1. How can the magnifying power of a telescope be found?
2. What difficulty may arise in the test?
3. How can the magnifying power of a microscope be found?
4. Can this be made to appear more or less, and if so, how?
5. How can the index of a given specimen of glass be approximately found?
6. What is a sine?
7. What is a radius?
8. What is an arc?
9. What is an angle?
10. What is the relation of dioptres of power to dioptres of curvature in ordinary glass?
11. In a dioptric medium whose index of refraction is 2?
12. In one whose index is 3?
13. In one whose index is 1.54?
14. Give the rule for finding the relation of curvature to dioptric power when index of refraction is given.
15. If the lens measure makes a lens a + 2, but the test with a far distant light makes its principal focus sixteen inches, what is its index of refraction?

CHAPTER XVI.

CYLINDERS.

There is considerable call in optometric practice for the use of a section of a cylinder such as is shown in B, Fig. 56; also for the reverse form as shown in C. Though according to the correct use of the word "cylinder" A is a correct representation of this form of a solid. Opticians, however, have come to accept a part of a cylinder as of the same name as the whole body, though the term cylindrical lenses is preferred by many. In both the forms of cylinders used, minus and plus, it will be noted

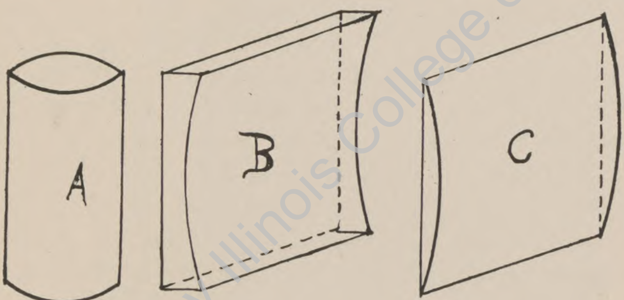

FIG. 56.

that there is one direction in which the surface has no curvature. This direction is called the axis of the cylinder. The direction of the curvature which has the greatest dioptric power at right

angles to the axis is called the meridian, or power, of the cylinder. In marking or expressing the power of a cylinder the dioptric power of the meridian is given, but the direction of the axis also must often be stated.

Cylinders are used to correct astigmatism.

Astigmatism is a refractive condition of the eye whereby the various meridians are not of the same dioptric power. This may be due to any of the following reasons: Inner surface of the retina not spherical in form; the curvature of the cornea not the same in all directions; the lens not of the same refractive index in all its parts, and there are other reasons. Whatever the cause may be, however, the difficulty is corrected by the wearing of cylindrical glasses so arranged as to power and direction of axis that the combination of cylindrical lens and eye, working together, produces a clear retinal image. To reach this result a minus cylinder may be needed on a certain axis; in which case the astigmatism is said to be myopic, or if a plus cylinder is needed the condition is said to be hyperopic. This applies to all those cases where no spherical lenses are needed, but the cylinders are sufficient. Sometimes a minus cylinder on a certain axis and the same power of plus cylinder on the opposite axis will both bring good sight. This is a result of ocular accommodation.

In dealing with cylinders the first thing to discover and mark is the direction of the axis. To locate this hold the cylinder between the eye and a long line. Turn the cylinder in the fingers, keeping it always perpendicular to the line of sight. Just as soon as the axis of the cylinder is in the same direction as the line, the latter, when observed at the same time over the cylinder, through it and below it, will appear unbroken. The apparent position of the line across the cylinder marks either

its axis or its power. To find which is which, move the glass back and forth in the direction of the line, also at right angles to it. In whichever of these two directions distant objects remain stationary this is the axis sought, but the direction in which there is motion; that marks the power of the cylinder. When the axis is found its direction should be marked on the glass by a line in ink.

The axis of a lens is marked on the basis of degrees. Let some person face the experimenter, and before the right eye of the former hold a cylinder. Imagine a horizontal line drawn across the pupil of this eye. The inner end of this line next the nose will be zero; the opposite end will be 180 degrees, and the intermediate values will count in the reverse direction to the movement of the hands of a watch; that is, 90 degrees will

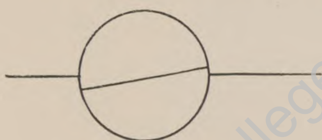

FIG. 57.

be at a medial point *above* the eye. The same direction applies to the left eye, but the counting must commence on the *temporal* side.

Fig. 57 shows appearance of lines along axis and appearance when axis does not coincide.

Cylinders in trial cases are usually marked as in Fig. 58. A shows axis at 90 degrees, B axis turned to 180 degrees, C axis at 120 degrees, D axis at 30 degrees.

Having found the axis of a cylinder the next point is to find its power. This may be done in a number of ways.

First—With a lens measure. Place the points of the measure in contact with the cylindrical surface, at the same time turning the instrument in the fingers so that the contact points may touch in turn all the meridians of the surface. The needle on the dial will show variations in power, marking zero at the axis; rising from this as the instrument is turned until the highest power is reached, and then once more descending. The point

FIG. 58.

where it marks zero will be the axis; where it shows the highest reading will be the power. Midway between the two extremes the needle will show one-half power; one-third of the way from zero to maximum it will show one-third power; two-thirds of the distance it will show two-thirds powers, etc., thus making clear that the curvature of the cylindrical lens increases steadily in power from the axis to the sharpest meridian.

Second—By neutralizing with glasses from the trial case. The first step is to find the direction of the axis in the manner already described, then select from the case some cylinder of the opposite sign. If a minus one is being tested take a plus cylinder from the trial case, or vice versa; place one upon the other with axes coinciding and move the two from side to side with the axes held vertical. If distant objects seem to move in the same direction as the two cylinders then the combination has a minus value, or if the distant objects seem to move contrary

to the direction of the movement of the cylinders then the combination has a plus value. In either case the cylinder from the case must be changed until no motion can be detected; in fact, there should be no motion no matter in what direction the couplet is moved. When this point is reached the value of the unknown cylinder will be the same as that of the one from the trial case; of the same axis, but of the opposite sign.

Third—By focal images. This method applies only to plus cylinders, and is practically the same as that used in getting the focus of spherical lenses. A cylinder, however, does not bring rays of light to a focus, but each point of the source of light shows itself on the screen as a very narrow line corresponding in length to the diameter of the lens being tested. The direction of this line will correspond with the axis of the cylinder. The light used should be as small and as bright as possible, and

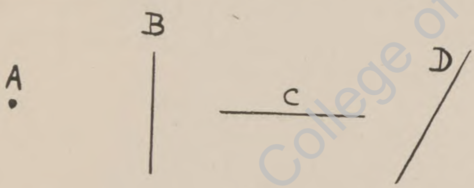

FIG. 59.

the cylinder should be moved back and forth until the narrow line of light formed on the screen is made the narrowest possible. This will be the focal point of the lens. If in Fig. 59 A represent the source of light B will show the appearance of the focal image if the axis is vertical, C if the axis is horizontal, D if the axis is at forty-five degrees.

Fourth—Minus cylinders may readily be measured by the method of reflected images as already explained in the case of

minus spherical lenses, but the light must be bright and at a distance. The minus cylinder should be moved back and forth from the card until the narrowest possible focal line is obtained, when the power of the cylinder may be calculated in accordance with the rule of one to four. It will be noted that the direction of the line image corresponds with the axis of the cylindrical surface.

Fifth—Minus cylinders may also be measured by the method of size of illuminated area, making allowance, of course, that the increase will be in one direction only, namely, parallel with the meridian of greatest power.

The principal focus of a cylinder is the focus of its meridian of greatest power. The conjugate and virtual foci of cylinders are figured on the basis of their greatest line of surface curvature.

A cylinder is sometimes called an astigmatic lens, meaning a lens which does not bring rays of light to a point, or without stigma, spot or point.

QUESTIONS.

1. Define a cylindrical lens.
2. What is the axis of a cylinder?
3. For what are cylinders used?
4. What is astigmatism?
5. Name some of the reasons for its existence.
6. What effect does the proper cylinder before the eye have on the retinal image?
7. How may the axis of a cylinder be found?
8. In what direction are the degrees of axis figured?
9. How is the direction of the axis usually shown in trial case lenses?
10. How may the strength of a cylinder be measured with a lens measure?
11. How by neutralizing?
12. How by focal image method?
13. How may minus cylinders be measured by the reflection method?
14. How by the method of increase in size of illuminated area?
15. Where is the principal focus of a cylinder?
16. What is the derivation of the word astigmatic?

CHAPTER XVII.

If we with a cylinder focus the rays of light coming from a distant bright point upon a screen it will be seen to be a narrow line parallel to the axis of the lens. If now the distant source of light be increased in size the narrow line on the screen will only be changed to the extent of making the line a trifle wider. If the light be changed to a line of light parallel to the axis of the cylinder there will be no apparent change in the image, which, in fact, is brightened, but so little as to be

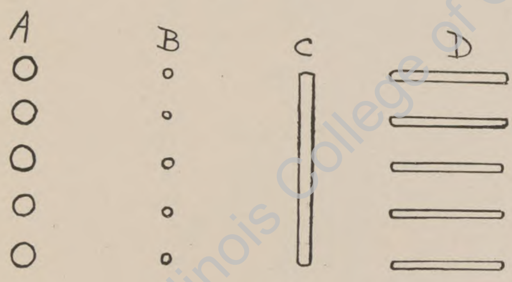

FIG. 60.

indiscernible. At first sight under these circumstances it would seem that the image was a correct copy of the original, not at all diffused or blurred, and yet it is a blurred image. This apparent anomaly can be cleared up by making the source of light not a line, but a series of points which together form a line.

With this arrangement if the image is a clear one it will also be a series of points forming a line, but this does not occur at all. Let Fig. 60 A show such a line of dots parallel to the axis of the cylinder. In this case the image will not be a line of dots as shown in B, but a line as shown in C, showing that the image C formed by the cylinder is really a number of straight lines, one for each of the dots in A, one superimposed on the other with the result that a single line of light is formed. Turn **cylinder so that axis is horizontal. The image formed on screen** is shown at D, each point of light appearing as a line but with the lines parallel and separated instead of being superimposed. From this it is made clear that when the axis of the cylinder

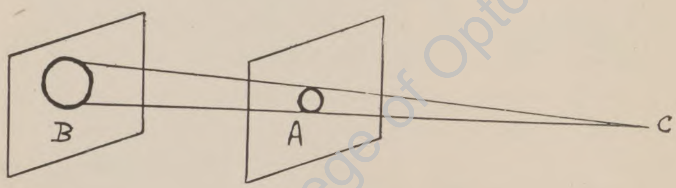

FIG. 61.

and the direction of the line of light are parallel the image of the cylinder will be a single narrow line because the many lines formed by the various points of which the line of light is made up lie one on top of another, while when the axis of the lens and the line of light are at right angles the image will be broad and dull, always in the direction of the axis, due to the fact that when the linear focal images are in the same direction they overlap and accentuate the light; while when the linear focal images are at right angles to the line of light they do not overlap; hence are comparatively dull. Fig. 61 shows a good way to

make clear the action of a cylindrical lens. Let C be a small source of light, let A be a piece of cardboard containing a central hole about one inch in diameter. Let B be a white screen. It will be seen by following the lines coming from C that there will be an illuminated area on B of about the same size as the hole in A. Now over the aperture in the card B place a cylindrical lens of any convenient dioptric strength, the axis horizontal, which is the same thing as saying that the power is vertical. Bring the screen B to the focus of the cylinder and the area of light on its surface will be brought to a horizontal line because the effect of a cylinder with power vertical is to cause vertical compression of light rays. Remove the cylinder and the illuminated area will once more agree with the hole in the screen A. Now place the power of the cylinder horizontal and the light area will be brought to a vertical line, because a cylinder with power horizontal is to cause horizontal compression. By considering axis instead of power we reach this rule: that a cylinder causes the image of a point of light to appear as a line of light parallel to its axis.

Taking as our source of illumination a cross of light we can get by a study of the images formed with a cylinder, an idea of the appearance of the clock dial lines in astigmatic eyes and what the meaning is of the phenomena observed. Let Fig. 62 show the illuminated cross, then with the axis of the cylinder vertical, the cross will appear as in the middle figure, and with the axis horizontal it will look as in the third figure. It will be evident from examination that the clearest line is parallel to the axis and that hence the defective meridian is in the same direction, since if we were to supply the missing power on this meridian the cylinder would become a sphere and the image of the source of light would be clearly defined. This

is an important point; namely, that the meridian which shows the clearest line is the one that is wrong; since it is the axis of the cylinder that lacks the necessary curvature to make the lens a

FIG. 62.

sphere, and it is also in the direction of this axis that the line is narrow, bright and not diffuse. The two points taken together make the rule clear.

The effect of minus cylinders is to diverge the rays of light at right angles to the axis. Supposing that the source of light in the case of these cylindrical lenses be a point, then with a lens as

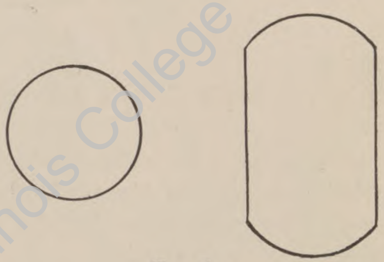

FIG. 63.

shown in Fig. 63 the image formed on the screen would not come to a linear form as is the case with plus cylinders, but would take an oval form, its length increasing as the cylinder is withdrawn, the direction of this length being parallel to the power and at right angles to the axis of the cylindrical lens.

Is it possible to so combine two cylinders that together they will correspond to a spherical lens? Let A in Fig. 64 be a plus cylinder axis placed vertical, and let B be another plus cylinder of the same dioptric value but with the axis placed horizontal. What will be the result from a point of view of refraction, to place one of these on the other? If the dioptric power of each be assumed to be three, then in A the dioptric power at M, being on the axis is zero, while in B in the same location the power is three dioptres; total three. At N in the cylinder A the dioptric

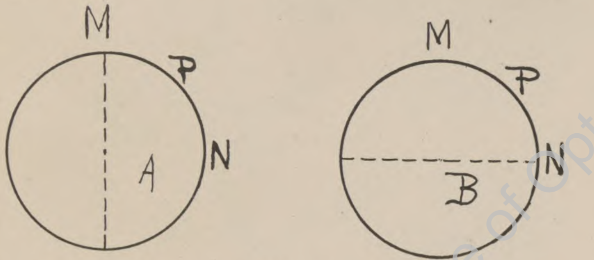

FIG. 64.

value is three; at N in B it is zero; total three. At P in lens A it is midway between M and N, hence must be one and one-half dioptres; at P in B it is half way between M and N, or one and one-half dioptres; total three; in fact, no matter what point we may select common to the cylinders the sum of their dioptric powers is always the same; therefore two plus cylinders of equal value with axes at right angles will be exactly equivalent to a spherical lens of the same dioptric power.

QUESTIONS.

1. What is the nature of the focal image of a point caused by a cylindrical lens?

2. What is the effect of changing the source of light from a point to a line?

3. Is this a blurred image, and why?

4. How can the true nature of the image be made clear?

5. With what apparatus may it be proved that a plus cylinder contracts light rays at right angles to its axis?

6. Which shows the defective direction, the bright line of the image or the dull?

7. What is the nature of the focal image of a minus cylinder?

8. How must two plus cylinders be arranged so that their combined powers will be equal to a sphere of the same dioptric value?

CHAPTER XVIII.

Sphero-Cylinders.

A sphero-cylinder is a lens equal in dioptric power to a combination of a spherical with a cylindrical lens. It is usually found with one surface ground to a cylindrical curve and the other to a spherical curve; though this is not universally the case, however, since some are made with both curvatures on the same surface, being known then as toric lenses. The advantages claimed for this latter form is that it can always be made periscopic with the minus spherical surface nearest the eye, whereas with the two styles of curves on opposite surfaces this is not always possible. To get an idea of the shape of a toric curve, look at the bowl of a spoon outside for the plus form, inside for the minus. It will be noticed that the curvature is not that of a sphere, that the radius of curvature is longest in one direction and shortest at right angles to this direction, while the curvature varies for every meridian between.

The result of a toric curve is the same as that of its equivalent sphero-cylinder; that is, it is always exactly equal to some combination of a sphere and a cylinder. Sphero-cylinders and their equivalent toric lenses are prescribed in those cases of astigmatism where simple cylinders are not sufficient.

The first point to learn of a sphero-cylinder of unknown value is the direction of its two principal meridians, namely, of

the meridians of least curvature and the meridian of greatest curvature. To get this, look at a long line through the lens, turning the same in the fingers until the line as seen both in the lens surface and out of it seems to be unbroken. This will be found to be the case in two directions, one at right angles to the other. These directions may be marked with ink. The meridian of lowest dioptric power is usually spoken of as the axis of the lens.

To measure out the value of a sphero-cylinder any of the following methods may be used:

First, by means of a lens measure. If the lens has the spherical curve on one side and the cylindrical curve on the other, the lens measure will show it; and the two readings can be combined. For example, if one surface moves the pointer on the dial to plus 2 in all meridians, and on the other surface moves it to plus 2 in the one hundred and twentieth meridian and to zero in the thirtieth meridian, then the dioptric power of the lens will be $+2 \frown +2$ cyl. axis 30. When we have to deal with a toric lens the lens measure will show on the sphero-cylindrical surface a minimum and a maximum reading, one being for the meridian of least curvature and the other for the meridian of greatest curvature; but usually a toric lens is periscopic, hence the curve of both sides must be allowed for. Suppose the toric surface reads $+6$ cyl., axis 45, $+6.50$ cyl., axis 135; and for the opposite side -4.00 spherical; what is the value of the lens? A $+6$ cyl., axis 45, $+6.50$ cyl., axis $135 = +6 \frown +50$ cyl., axis 45. Combine with this the -4 sphere, and we have as the net result a $+2 \frown +.50$ cyl., axis 45 degrees.

Second, by neutralizing. Hold the lens to be tested between the eye and a long straight line, turn in the fingers till this line both as seen in the lens and out of the lens at the same time will appear as an unbroken line, which will be in two directions, both

crossing the optical center of the lens, and one at right angles to the other. One of these directions is the line of least dioptric power; the other is the line of greatest dioptric power. Their direction may be marked with ink on the glass. Move the lens in a direction across the line, and estimate the dioptric power of either of these meridians; next place in contact with the lens to be tested one of the lenses from the trial cases of the estimated power; try if there is apparent motion of distant objects when the two lenses are moved together in the direction of the meridian selected. If there is, try another sphere of greater power if the motion is with the direction of the lens, of less power if the contrary is the case. Continue these trials till one meridian is neutralized; that is, till it shows no motion. Now estimate the additional cylindrical value of the opposite meridian and apply minus or plus cylinders with axis along the meridian first neutralized until there is no apparent motion in any direction. Note the value of the two lenses used to neutralize. Their value with signs reversed, but axis of cylinder unchanged, is the required dioptric power. For example, if there be required to neutralize the sphero-cylinder, or toric lens, a +2 sphere, and a —1 cyl., axis 120, then the value of the tested lens is $-2 \smile +1$ cyl., axis 120. It may be in the case just stated that the result would be $-1 \overset{\frown}{\smile} -1$ cyl., axis 30, but this is merely the first finding transposed. They both have exactly the same dioptric value.

Third, by focal images. This method can only be used in those lenses whose formula is either a plus cylinder on a plus sphere, or one which by transposition can become a plus on a plus. Hold such a lens between a distant small bright light and a white screen of any kind. Hold the lens close to the screen, and then gradually withdraw it until the image shows a fine line, the finest possible; measure the distance and express in dioptres. Repeat

the operation, but withdraw the lens beyond the first point and until another narrow bright line is formed at right angles to the first. Measure the distance of this from the screen and express it in dioptres. The first value will be the meridian of greatest power, and the second value will be the meridian of least power. The image formed by a sphero-cylinder varies in the manner shown in Fig. 65 according to its distance from the screens.

Commencing with the lens held close to the screen, the illuminated area will be of the same shape of the lens. Supposing the

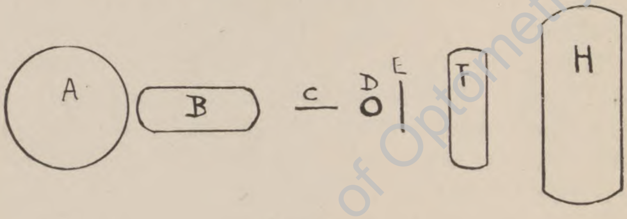

Fig. 65.

vertical meridian is the strongest, the bright area will contract more rapidly vertically than horizontally, so that soon the area will have the form B; this same process continuing further it will reach its limit when the focal image becomes the line C. Still farther away the lens, being beyond the first focal point but not yet to the second line, will contract still more horizontally, but since the rays are now crossed it will broaden vertically until the form D shows itself; still further away the horizontal contraction will reach its minimum limit and become a line which will be longer vertically than D, because the vertical rays have diverged still more. From now on both directions of rays show divergence, the vertical having the start; hence we soon have the oval F. From this point the area grows

larger and larger, but the actual difference between the divergence of the two directions remaining the same, the relative difference becomes less and less, so that the illuminated area as it grows in size approaches nearer and nearer to a circular shape.

The principle of measuring the power of minus lenses by the size of the illuminated areas they cause can be applied to the measurement of minus sphero-cylinder lenses as well, but a little care will be needed. Also where both surfaces are minus they can be measured by the reflections on their surfaces in the manner already described for minus spherical and minus cylindrical surfaces.

The appearance of the image of an illuminated cross made by a sphero-cylinder lens is worthy of considerable study, since the image formed in such case is very similar to the retinal image of the lines of greatest and least clearness of the clock dial chart in the case of astigmatic eyes.

Let Fig. 66 be an illuminated cross, the vertical arm marked A and the horizontal arm B. Form an image of this cross on a screen with a sphero-cylindrical lens held at the focal distance of its strongest meridian, this meridian to be vertical. Since this vertical meridian is at its focal distance, the horizontal one is not, since the latter is too weak. Under these conditions every infinitesimal particle of the cross will take the form of a horizontal line, this being the direction of the refractive deficiency, and the cross will appear as shown in the end drawing of Fig. 66. Here the relation between defective meridian and bright, clear line will come out. The image being made of an infinite number of short horizontal lines, all the points of the horizontal line of the cross will overlay and therefore seem bright, with clear edges, while all the points in the vertical line will be broadened out, and covering more space must be relatively dull.

If the sphero-cylinder now be withdrawn until the meridian of lowest dioptric power is in focus, under this condition the vertical meridian will be the defective one; all the infinitesimal parts of which the cross is composed will be focused as short vertical lines.

FIG. 66.

These in the image of the cross will overlie in the vertical direction but side by side in the horizontal so that the image will appear as the middle drawing in Fig. 66, thus again showing that the defective meridian is the one that shows the brightest line.

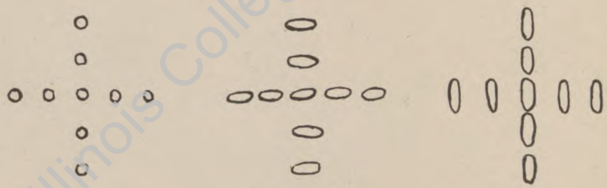

FIG. 67.

It is unfortunate that cylinders have come to be spoken of by the direction of their axes. Their effects, both as simple cylinders and sphero-cylinders would be more easily explained and understood if they were marked and referred to by their powers. If this were so we would correct an error in focus by placing the dioptric

power of the lens along the defective meridian. Now we have to say to place the axis along the opposite meridian.

If we make the cross of two series of small lights, as shown in Fig. 67, the effect of sphero-cylinders can be more clearly shown.

If the vertical meridian is in focus, then all the small lights will be lengthened horizontally and the image will appear as in Fig. 67, middle cut; the horizontal line being composed of overlapping images, showing clearly that the narrowest line in the cross marks the direction of the diffusion.

When conditions are reversed so that the vertical meridian is the defective one then the image will appear as in Fig. 67, end cut, in accordance with the rule above stated.

The above can be rather strikingly shown by making a cross of colored sections as shown in Fig. 68.

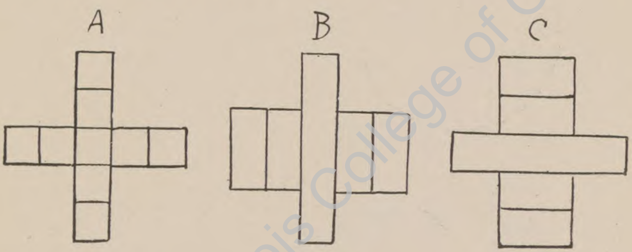

Fig. 68.

Look at such a cross through a strong cylinder. When the power of the cylinder is vertical the result will be something like Fig. 68 B, where the vertical broad line will have all the colors run together while the horizontal sections will be distinct one from another.

Reverse the direction of the cylinder so that the power is horizontal and the retinal image will be somewhat as in Fig. 68 C,

where the blending will be in the horizontal direction and the distinctness of separation of colored sections will be vertical. Here again it is plain that the line with clear edges and narrowest width marks the defective meridian; that is to say, the meridian which if corrected would cause the image to come right.

A sphero-cylinder is always equivalent to two cylinders of different values with axes at right angles; or conversely two crossed cylinders of two different dioptric powers with axes at right angles are equivalent to some sphero-cylinder. Let Fig. 69 represent the two cylinders, one of which is to be superimposed on the other.

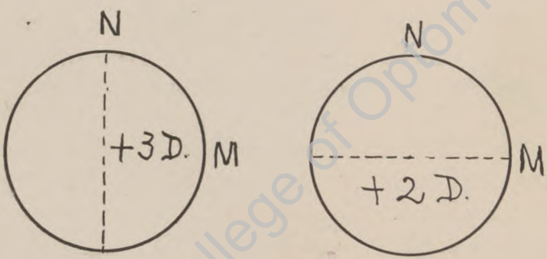

FIG. 69.

Let the first one be of three dioptres, axis placed vertical, and let the other be of two dioptres axis placed horizontal. At the point M the first glass has a dioptric value of +3; at the corresponding point M of the second cylinder there is no power; so that the value of the two together in 3 dioptres. In the same way the dioptric power of the two at N will be found to be +2. with a steady graduation from one to the other. This may be expressed in another way, namely, that there is a dioptric power all around the meridians of the lens with an extra power starting from zero at N and increasing to one dioptre at M; but the first of these is

the quality of a +2 sphere, and the second is the quality of a +1 cylinder axis 90; therefore the two cylinders correspond to the combination of sphere and cylinder as given.

Every sphero-cylinder is always equal to some other sphero-cylinder in which the cylinder axis is reversed, but the value in dioptres must remain the same. Changing one form of a sphero-cylinder to the other equivalent form is called transposition. There are two rules for transpositions: One for changing sphero-as follows: For the new sphere add the algebraic value of the old sphere to the value of the cylinder.

The rule for changing sphero-cylinders to sphero-cylinders is as follows: For the new sphere add the value of the old sphere to the value of the cylinder.

For the new cylinder reverse the sign and axis of the old one.

It must be remembered that no transposition of sphero-cylinder to sphero-cylinder can change the numerical value of the cylinder. Only its sign and axis can be changed.

EXAMPLES.

+ 2 D ◯ + 2 D. cyl. ax. 90 = + 4 D. — 2 D. cyl. ax 180
+ 3 D. ◯ — 150 D. cyl. ax. = + 150 D. + 150 D. cyl. ax. 120
— 2 D. ◯ — 1 D. cyl. ax. 15 = — 3 D. + 1 D cyl. ax. 105

Rule for changing crossed cylinders to sphero-cylinders.

Express one of the cylinders as the new sphere.

Deduct the value of this same cylinder from the value of the other cylinder. The algebraic difference will be the power of the new cylinder.

The axis of the new cylinder will be the same as that of the first cylinder above mentioned.

EXAMPLES.

+ 2 cyl. ax. 30 + 3 cyl. ax. 120 = + 2 ⊂ + 1 ax. 30
— 2 cyl. ax. 90 — 4 cyl. ax. 180 = — 2 ⊂ — 2 ax. 90
+ 3 cyl. ax. 15 — 1 cyl. ax. 105 = + 3 ⊂ — 4 ax. 15
+ 1 cyl. ax. 60 + 4 cyl. ax. 150 = + 1 ⊂ + 3 ax. 60

Another rule: Call the first cylinder X and the second one Y. Then the value of the new sphere will be X; the value of the cylinder will be Y — X, and the sign of the cylinder will be that of X.

QUESTIONS.

1. What is a sphero-cylinder?
2. What is its usual form?
3. Describe a toric lens.
4. When are sphero-cylinders prescribed?
5. How are the principal meridians of a sphero-cylinder found?
6. How is the value of a sphero-cylinder measured with a lens measure?
7. How is the value of a toric lens measured with a lens measure?
8. Describe measurements of the above by means of neutralizing.
9. How may they be tested by focal image methods?
10. Describe the shape of the diffused images formed by sphero-cylinders.
11. Describe some other methods of measuring these lenses.
12. Which arm of the image of a cross is blurred when the vertical meridian of a sphero-cylinder is out of focus?
13. When the cross is made of two series of lights at right angles, what is the appearance of its image when the horizontal meridian of a sphero-cylinder is in focus?
14. Describe the appearance of a cross made of blocks of colors when viewed through a strong minus lens.
15. Give the rule for the transposition of one sphero-cylinder to another of equal dioptric value.
16. Give an example.
17. Give one of the rules for the transposition of crossed cylinders to a sphero-cylinder.
18. Give some examples.

CHAPTER XIX.

PRISMS.

A prism is one of the forms of a solid in which there are two parallel sides, either one of which may be the base. If the prism is circular in section it is then a cylinder; if square in cross-section, it is a square or right prism; if triangular in cross-section, it is a triangular prism. (See Fig. 70.)

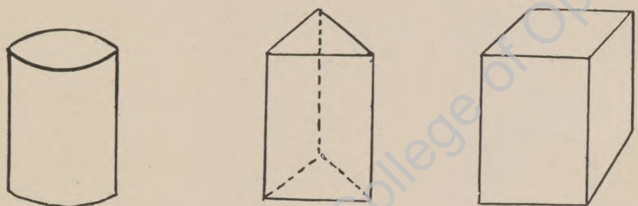

FIG. 70.

The prism used in refractive work is shown in Fig. 71. It is really a wedge of glass, more or less thin, in which the base is miscalled, this being really one of the sides of the prism, though it is the base of the wedge. In Fig. 71 the base geometrically is Y, but in optical practice it is the narrow rectangular side of the prism, and is opposite the sharp edge which is called the apex of the prism. This sharp edge, or apex of the prism, is at the joining point of the two larger and equal sides. (N. W.)

Any line from the apex to the base of the prism at right angles to each is called the base-apex line. (P.)

In geometry a circular prism is really a cylinder, while a square prism is a rectangular column, as shown in Fig. 72, but

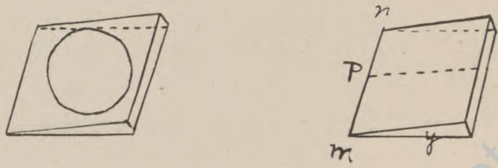

FIG. 71.

in ocular work a round prism and a square prism are the same wedge of glass, but in one case the outline is square, while in the other it is cut away to a circular form so it may be conveniently used in a trial frame. (See Fig. 73.)

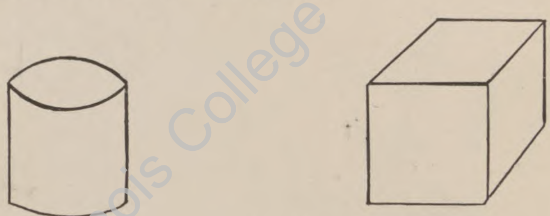

FIG. 72.

In the case of the round prism shown in Figure 73, it will be seen that the apex and base are both cut away so that the base-apex line can only be seen in its entirety in one point shown at M and N, while with the square prism it may be seen at any point between A and B. This makes it hard in using round prisms to know the exact location of this base-apex line, since it can only be

judged by the thinness of the edge of the point M, but the thinness of the glass at this point is so very little different from that of other points near it that an error is quite easy. In the case of square prisms this is not the case, since the base-apex line is always parallel to the edges A B and C D of the larger sides. It is on account of this difficulty that the most expensive trial cases have both round and square prisms—the former to use in the trial frame, the latter for the hand, with which they can be quickly tried without the necessity of looking on their surface

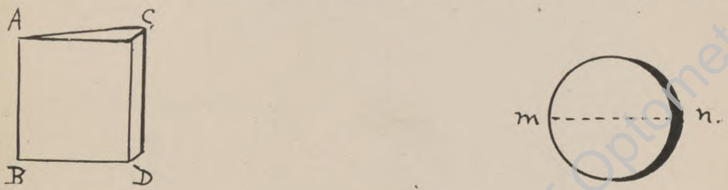

FIG. 73.

for the mark which is put there by the manufacturers to show the base-apex line.

The base-apex line of a prism may be located or verified by looking through it at a long line. Hold the prism approximately in position and then give it a circular motion in the fingers. At the same time watch the line in its entirety, not only the part of it which appears through the prism, but also that part of it which shows under and above the glass. Under this test the portion of the line seen through the glass will take one of the positions shown in Fig. 74, the apparent distance of X from the long line varying with the extent to which the prism is off axis, but the direction of X (Fig. 74) will always be parallel to A B. The line X will never take the slanting position so characteristic of cylinders.

If a ray of light from any source be traced through a prism it will be found to have been bent in such a way that its new direction is always more toward the base of the prism than its old, while the eye which receives this ray will seem to see the object in the reverse direction, namely, toward the apex.

Let Fig. 75 be such a ray of light passing through a prism. Let N be the prism, M the source of light, Y the apex of the

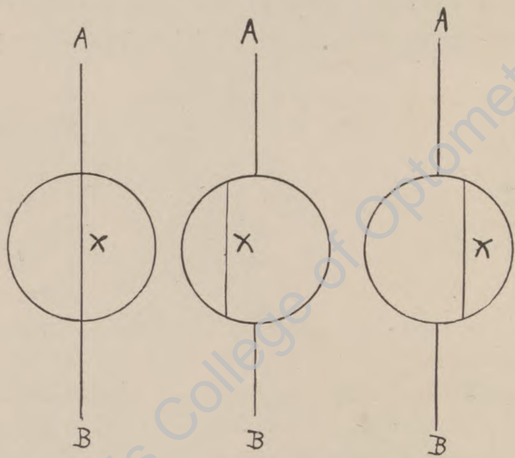

FIG. 74.

prism, C the location of the eye. As the result of the refractive power of the prism the ray from M will emerge in the direction of C, but an eye at this latter point would seem to see the object in the direction of X, (since the eye always assumes that the object viewed is straight out and not around any corners.) The eye in placing the object in at X must turn toward the apex of the prism; that is, if the eye look directly at M without a prism inter-

vening and then a prism be brought into view the eye, in order to bring the image of M to the ocular yellow spot it must turn its line of sight toward the apex of the prism. The line which joins the eye with the apparent location of the object is called the line of projection. The term projection is used considerably in scientific optics, and means that function of the eye by which an image on the retina is always ascribed to that point in space which is the conjugate focus of the yellow spot, no matter whether the object is there or not. The main fact is the retinal image, and from its location the position of the object which under normal cases would produce it is assumed.

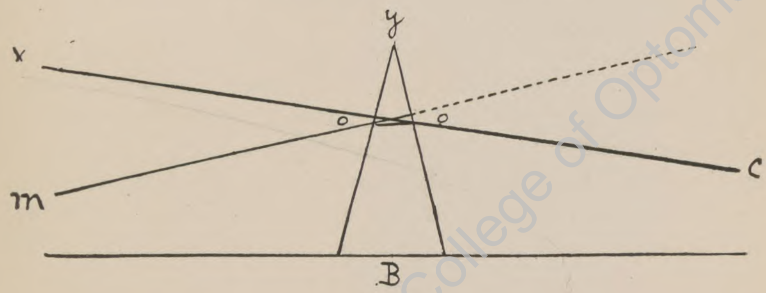

FIG. 75.

The angle of a prism is the angular opening of the two sides of the prism where they come together in the sharp edge; that is, at the apex. Prisms are usually numbered by their angles, though there are other standards based on the power of the prism to bend or refract the rays of light.

The extent to which a prism refracts a ray of light is called the angle of deviation. It varies somewhat according to the angle of incidence, but not enough to be regarded in ocular prac-

tice; also it varies according to the angle of the prism. For weak prisms the relation of the angle of deviation to the angle of the prism is about 1 to 2, so that a six-degree prism will correspond to three degrees of deviation. For larger powers this rule is not so correct, but near enough for all practical purposes. In Fig. 75 O shows the angle of deviation that is the difference in direction of X C and M.

To test the strength of a prism, and therefore to get its angle, the first thing necessary is a working scale, the dimensions of which will depend upon the distance selected to make the test. Suppose we settle upon ten feet as a convenient distance; then it will be necessary to draw upon a white A card a number of long parallel lines, the spaces of which must correspond to a tangent scale on the basis of one degree. The distance of these spaces apart may be figured from any list of natural tangents, but for the strength of prisms used in ocular refraction this refinement of measurement will not be necessary, and the scale can be figured on the basis of the equal one degree divisions of the arc of a circle. In fact, the error of this approximate method will not be discoverable and is very apt to be less than errors due to incorrect manufacturing, slight as these may be.

Ten feet being the chosen distance, imagine a circle of this radius. This will be a diameter of twenty feet. Multiply this latter by 3.14156, as may be obtained from any geometry, first, however, expressing the twenty feet in inches. Since there are three hundred and sixty degrees in any circle and ten degrees are one-thirty-sixth of a circle, divide the product obtained above by thirty-six and the quotient will be the length of a ten-degree arc on a ten-foot radius. This will be approximately twenty-one inches. Make the number of lines on the white card eleven; this will be ten spaces, each one of which is the arc of one degree.

Now draw between each two of these lines other lines exactly midway and each of the spaces will be the arc of one-half a degree, which is so close to the value of the tangent of one-half degree that the two may be considered as identical. Number these lines as shown in Fig. 76.

The experimenter should take a position eleven feet from the scale and hold the prism one foot from his eye, which is ten

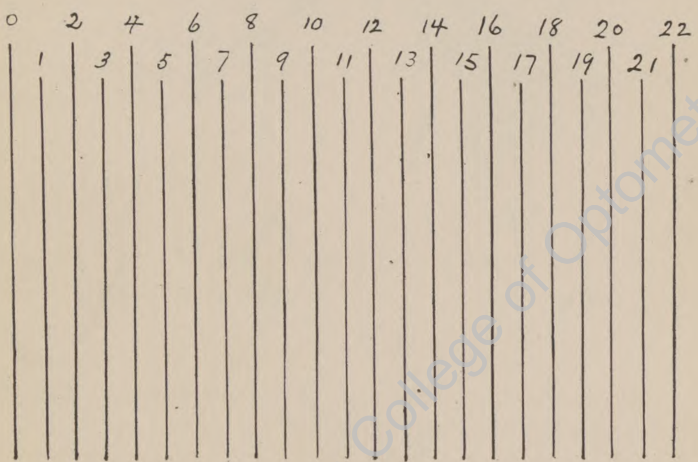

FIG. 76.

feet from the scale, with the base-apex line previously marked horizontal. Then looking through the prism at the lines and at the same time looking over and under it the lines will appear broken. Fix the attention on the zero line; notice the extent of its misplacement in the prism, reading by the figures on the scale. The line on the scale which seems to coincide with the zero line is the measure of the angle of the prism in degrees. The space

on the scale really represents one-half a degree, but the test shows the angle of deviation, and the angle of the prism as above stated is twice this; hence the scale spaces represent degrees of angular difference in the prism. If the appearance of the lines is as shown in Fig. 77A, then the angle of the prism is three degrees. If as shown in Fig. 77B, then the angle of the prism is seven degrees.

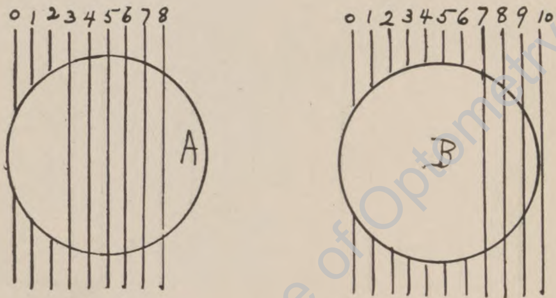

FIG. 77.

If it is desired to work at twenty feet instead of ten the width of the spaces must be doubled, and so on for any distance.

The same thing can be reached approximately in a shorter way requiring no scale. Look through the prism at anything with a straight edge, like the test card; turn the prism in the fingers until the greatest displacement is shown; then estimate this in inches. For a distance of ten feet each inch of displacement will be about one degree of prism angle. If the distance is twenty feet the displacement per degree will be about two inches.

To measure out sphero prisms, first find the value of the sphere by any of the methods suitable for this purpose; then find the

greatest displacement by turning the lens, but making sure that the geometrical centre of the lens is tested and read off the prism value on the scale. It is assumed that a lens is always designed so that the eye looks through the geometrical centre, and to know where this is being tested in a prism move the prism from side to side while testing till the direction of the zero line on the scale cuts the geometrical center, not the zero line

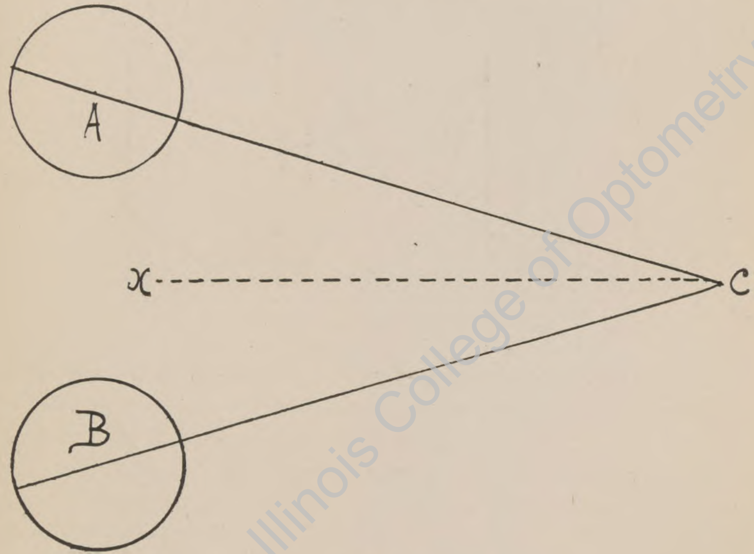

FIG. 78.

itself, as seen in the prism, but the place where it would be if the zero line were unbroken. In a sphero prism if the geometrical point is not in the line of direction tested the result will be incorrect, as decentered lenses mean a prism value dependent

upon the amount of the decentering and the dioptric value of the spherical element of the sphero prism or simple sphere, as the case may be.

To measure out cylinder prisms or sphero-cylinder prisms is not so easy, especially where the axis of the cylinder and base-apex line of the prism cross obliquely, but it can be done in

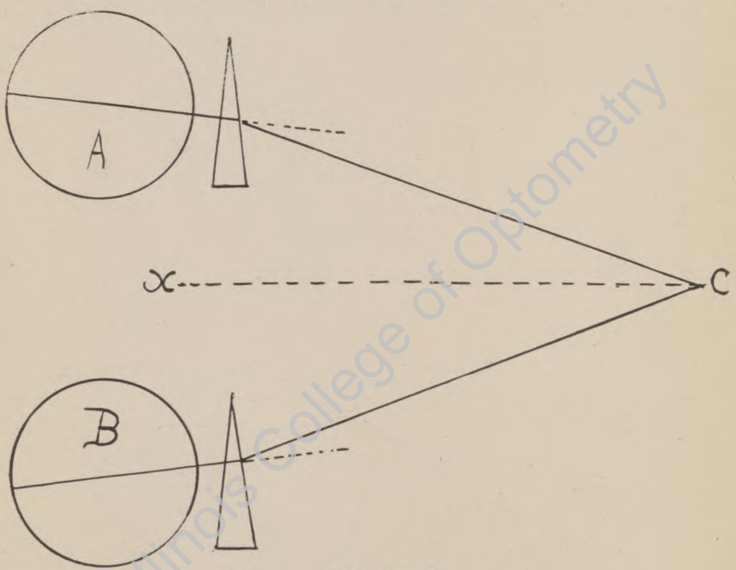

FIG. 79. *

accordance with the methods already laid down, a successful result being mainly a matter of patience and care.

In the use of prisms some curious phenomena are apt to occur because of the interference of the changed direction of line of sight with old habits. Let us suppose that a person with

*NOTE.—The lower prism should have been drawn with its base upward instead of downward.

normal eyes is looking at a distant object as shown in Fig. 78. It is evident that the direction of the object seen is along the line X C, running from the point midway between the eyes to the object; also that in looking at the object there is a certain amount of turning in of the eyes, or convergence, to bring the image of the object on the yellow spots of both eyes.

Now place before the two eyes, as shown in Fig. 79, two prisms with bases in. Since an eye in looking through a prism always turns toward the apex the two eyes will now look straight ahead toward X and Y. This is what takes place normally when we look at objects far away, so that we have come to associate the two; hence in the case as shown in Fig. 79, in spite of the fact that we know that the object C is near, our brain insists on acting in accordance with old habits and will see it far away. At the same time as the brain judges of the size of an object to some extent by the size of its retinal image the object C will also look disproportionately large. Should the prisms be used base out the opposite result would take place; the object would seem too close and too small.

Let Fig. 80 be similar to Figs. 78 and 79, but with one prism only in place, say base in over A as shown. Here one eye look straight at the object while the other looks around the prism corner. In this case the eyes are directed somewhat wider than is normally correct for the point C, hence the object will seem too far away and too large; also since the eye sees the location of an object midway between the direction of the line of sight of the two separate eyes, therefore the object will be seen misplaced in the direction of Y. Put the prism on the other eye, base in, and the object will be seen at Z and larger than it really is. Reverse the direction of the prism and the object will still be misplaced, but will seem too near.

It is this interference of prisms with ocular habits that is the cause of so much annoyance to those for whom they have been prescribed, and, in fact, so serious are the objections made for them that the practitioner has learned to be far more cautious and conservative in recommending their use than was the case a few years ago. The patients complain that the steps of the stairs look far away or seem too close, so that they have difficulty

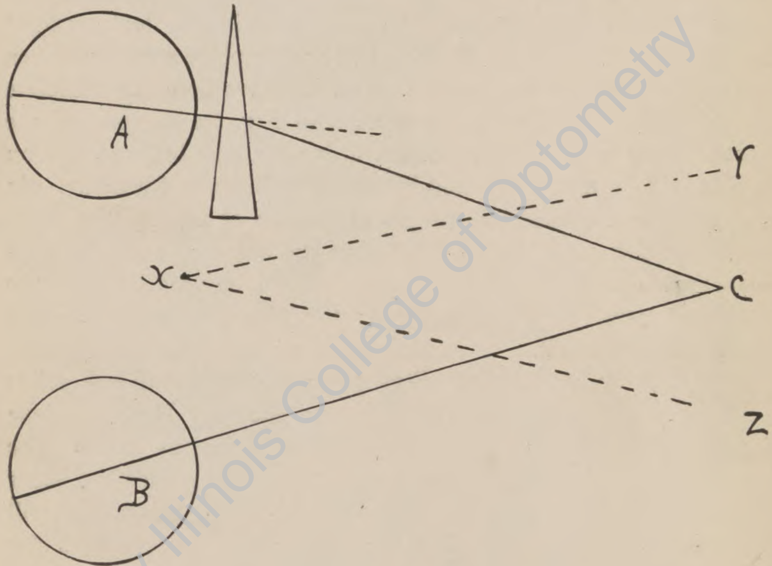

FIG. 80.

in avoiding a fall, or that they run into obstacles that are much nearer than they look, or that they dodge the same much to the amusement of the bystanders when they are really several feet away, and many others of the same kind.

As to the use of prisms for muscular troubles, that is a point of argument for the specialists.

QUESTIONS.

1. What is a prism as defined in geometry?
2. What is a prism as defined in ocular refraction?
3. Define the base of a prism.
4. What is the apex?
5. What is the base-apex line?
6. Tell what you know about square and round prisms.
7. What is the particular advantage of each?
8. How can the base apex line of an unmarked prism be found?
9. What does a prism do to a ray of light?
10. In which direction does the eye move in looking through a prism?
11. What is the angle of the prism?
12. What is the angle of deviation?
13. What is the line of projection?
14. What is the relation of the angle of deviation to the angle of the prism?
15. How is a scale made for testing prisms?
16. How is the prism tested with such a scale?
17. How can a quick estimate be made of the angle of a prism without using a scale?
18. What is the effect of decentering a lens?
19. How can sphero prisms be measured out?
20. Why do prisms make objects seem of the wrong size and distance?
21. What are some of the reasons given by patients objecting to the wearing of prisms?

CHAPTER XX.

DISPERSION.

It is evident that since the various colors have different indices of refraction that if we can make the bending of a beam of white light at any point strong enough that these colors can be made to separate. This is readily done with sunlight and a prism of high angle size, say one of sixty degrees. When sunlight falls upon such a prism, preferably a narrow beam of the sunlight, and a white screen is held in the right place to catch the refracted light a most gorgeous apparition will appear—a dazzling display of color arranged as in the rainbow. This is called a spectrum and shows clearly that light is a mixture of colors. Such a process for separating light into its constituent colors is called dispersion.

The colors of the solar spectrum are red, orange, yellow, green, blue and violet, but with all gradations of shades between. If it were not for the fact that each color has a different index of refraction the composite nature of white light might still be unknown.

In Fig. 81, let B C be a narrow beam of sunlight; then the red rays in this beam which is the least refrangible would appear at D, and that point would reflect red light accordingly; but the violet rays being the most refrangible, or what amounts to the same thing on the basis of the wave theory, being the most re-

tarded of all the visual rays, would be bent more sharply and would be caught on the screen at the point F. The yellow producing rays having a refrangibility between the two would appear on the screen midway between D and F. The result of all these variations in the refrangibility of the various colors results in the spectrum.

This dispersive property is possessed by prisms of all sizes, but to be well observed prisms of high angular value are best

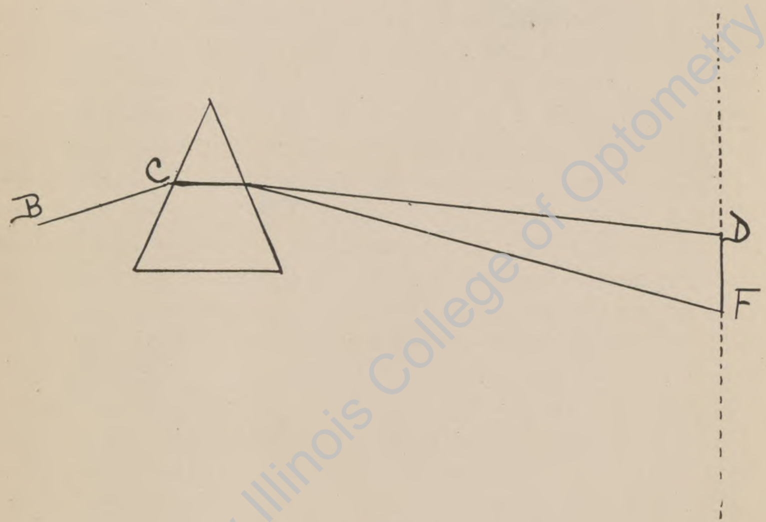

FIG. 81.

as they produce the effect in the most marked degree. The space on the screen occupied by the colors of the spectrum is called the visible spectrum and extends from deep violet to the deep red. There is far more extent than this to the spectrum though

the eye cannot perceive it all. There is an invisible spectrum shown by the dotted lines in Fig. 81 which is several times longer than the visible spectrum. The existence of this on the screen can be readily detected. For instance, suppose we take a delicate thermometer and starting at the middle of the visible spectrum we move it gradually in the direction of the red end, and then beyond we will find the heat steadily increases to its maximum, which is beyond the visible red end, from which point as we move farther and farther away it becomes less and less. This experiment carefully conducted shows two important facts—first, that heat exists a considerable distance from the red end of the spectrum, and that the greatest amount of it is at a point on the screen where there is no light at all. If the opposite end of the spectrum is tested the heating effect soon disappears; if we take a large number of photographic plates and test the photographic intensity of the different points of the spectrum we will find that the red end has little or no effect. It is for this reason that photographers develop their plates by a ruby-colored light. If, however, we test the other end of the spectrum we will find an extremely violent photo-chemical effect beyond the violet end, and, in fact, we can trace in this way the invisible spectrum for a long distance beyond the violet end.

The interesting question arises, what relation, if any, has the visible to the invisible spectrum? Is it formed by some different form of energy, or is it of the same general character as light? To get some clear ideas on this question the simplest plan is to try to subject all the parts to the same general tests. If all parts of it react in the same manner to these tests it will be a safe working hypothesis to accept as our belief that the three forms of energy are all varieties one of another. To this end suppose we try to reflect it with a mirror, either plane or concave.

Hold a thermometer to one side and arrange a mirror at such an angle that if there are rays coming from the prism they will be reflected upon the instrument. The result will be heat, showing that the invisible rays can be reflected. Repeat the experiment with a lens and the result will show that the rays can be refracted. Next repeat the experiments at the opposite end of the spectrum, in the ultra-violet region, using a photographic plate as a means of test. The result will be the same. This portion of the invisible spectrum can be reflected and refracted.

It is evident that the sunlight is very complex in nature and that it is owing to the varying refrangibilities of its elements that we are able to detect these.

Although a prism will permit the various waves which make up the visible and invisible spectrum to pass through its substance this is not true of all substances. No two of them produce the same result, some being opaque to light, some to heat, some to chemical effect, some to mixtures of these, and no two alike.

The effect of a piece of red glass is to absorb all the rays of light except the red which passes on through. The effect of a piece of green glass is to pass only green—to absorb the rest. Therefore it would seem that the two together would shut off light altogether, since if the red absorbed all but this color nothing else would pass, and since green was transparent to this color only and the red had already shut off the green there ought not be any left at all. If it were possible to find glass which allowed red only to pass and another glass which permitted only green to pass this result would come true, but no such glass exists. What we really have is glass in which one color passes freely, but which also allows small portions of other colors to pass also. If the colors passing one kind of glass are much red and a little blue, and the colors passing another sample are much green and

a little blue, the result of the combination will be that the green and red will shut off one another, but the blue will pass both. The same reasoning may be applied to painters' colors, which when mixed do not show a combination of the two colors, but rather the colors that are left after the two pigments have exercised their absorption powers. For instance, chrome yellow and ultramarine blue make green, while the colors yellow and ultramarine blue make something very close to white.

To show the effects of combinations of colors the apparatus shown in Fig. 82. Let B (Fig. 82) be a patch of color, let C

FIG. 82.

be another, let A be a vertical plate of glass, let D be the location of an observing eye. Under these conditions the eye will see B through the glass and C by reflection, and the two colors will fall on the retina at the same time. This is something entirely different from mixing colors.

If sunlight be made to pass through a tank containing a solution of alum all of the heat rays will be absorbed and disappear. If the light pass through a solution of iodine all the

light rays will be absorbed, but the heat rays will pass. Let the rays pass through both, and both heat and light will be absorbed.

In Fig. 83 let C be a beam of sunlight, A a glass tank containing solution of alum, which is transparent to light, and B a glass tank containing iodine solution, which is transparent to heat. Let L be a double convex lens of high dioptric power, of which F is the focal point. Hold a piece of white paper at F and there will be no image, since the iodine tank cuts off all the light rays.

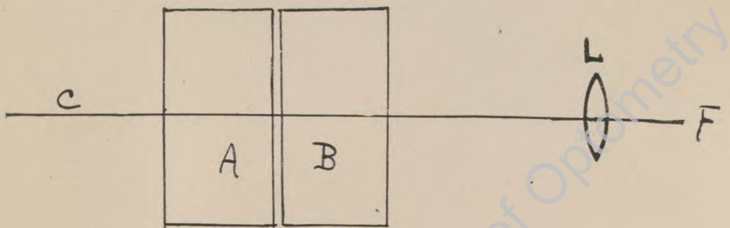

FIG. 83.

Remove the iodine tank, leaving the alum tank, and repeat. An image will be formed just as bright and clear as though the tank were not there. Remove the alum tank and put the iodine tank back. Now there will be no image, but a match will take fire at the focus, showing that heat rays have passed. A test of the amount of heat with a thermometer will show that there is a great deal of it there. Finally, remove the iodine tank and substitute the alum tank. Now light will pass no heat, and the match will not burn.

As the result of experiments of this and of a similar nature with all forms of energy, scientists have come almost to accept the belief that all rays may be grouped under the one term—radiant

energy, to which it is almost certain that not only light, heat and photo-chemical action belong, but also electricity, magnetism, the X-ray and the latest discovery, the energy radiated by radium.

QUESTIONS.

1. What is dispersion?
2. Describe the solar spectrum.
3. How can the spectrum be produced?
4. Name the two general divisions of the spectrum.
5. Where in the spectrum are the heat effects the strongest?
6. Where are the light effects strongest?
7. Where are the photo-chemical effects the strongest?
8. What similar properties have the visible and invisible rays which make up the spectrum?
9. What is the difference between combinations of pure colors and combinations of pigments?
10. How can we cause the retina to receive two colors at the same time?
11. What is radiant energy?